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Proposed Modified Clarke-Wright Saving Algorithm for Capacitated Vehicle Routing Problem

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ABSTRACT

A multi-objective distribution routing algorithm by using modified Clarke and Wright Saving algorithm is presented. The problem to solve is to deliver loads to a number of outlets based load requirement. The objective function to minimize is the distance saving and traveling time of the resulted route started from depot to the outlets and return to the original depot. Problem to solve is generating a distribution route in a week considering traffic condition for each day. The original Clarke and Wright saving algorithm is modified such that the resulted routes (from a depot to some outlets) accommodates some constraints such as the maximum allowable traveling time, maximum number of delivery shifts, and maximum number of vehicles. The algorithm is applied to a distributor company with nine outlets, two vehicles, and two delivery shifts. In addition, the traffic condition on the outlet-to-outlet and the depot-to-outlet routes is considered. The simulation of the proposed algorithm shows that the algorithm can generate routes that comply with shift's maximum delivery time and the vehicles' capacities.

Keywords: Clarke-Wright saving matrix, capacitated vehicle routing problem, optimization, traveling time.

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1. INTRODUCTION

Historically, the term "vehicle routing problem" was initially known as "truck dispatching problem", which was introduced in Dantzig and Ramser (1959). Initially, the problem was about finding the shortest-distance way for a fleet of homogeneous trucks such that they can deliver a set of gas stations from a central depot. As the problem in the real world has become more complex, variants of VRP were introduced. One well-known branch of VRP is the capacitated vehicle routing problem (CVRP), which is defined as the problem of finding optimal routes by visiting a set of nodes by considering the maximum load capacity of the vehicles.

2. LITERATURE REVIEW

A number of CVRP variants have been introduced. For instance, Pichpibul and Kawtummachai (2013) presented the CVRP with single depot and the optimal solution had to minimize the transportation cost. Another variant considering traffic jam was presented by Mandziuk and Swiechowski (2017) and Sabar et al (2019). Fuel consumption, which in turn affects the production of CO_2, becomes a part of objective function in Xiao and Konak (2016) and Matos et al. (2018). A variant of CVRP with multiple depots and time windows were reported in Bae and Moon (2016). Here, the nodes to visit were categorized into the delivery nodes and installation nodes. Therefore, the delivery and installation times became components in the cost function. The use of CVRP to handle the problem of two-echelon distribution systems was reported in Li et al (2015) and Li et al (2016). Here, a set of central depots have to deliver goods to a number of satellite depots before the goods are delivered to the customers.

Since the CVRP is an NP-hard problem, mostly the

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methods used to find the optimal routes are of the class of meta-heuristics ones. From our investigation, genetic algorithm (GA) was the popular meta-heuristic method (Bae and Moon, 2016; Mandziuk and Swiechowski, 2017). Iterated Local Search (ILS) was used in Matos et al. (2018), and Upper confidence bounds applied to trees (UCT) was used in Mandziuk and Swiechowski (2017). The use of MILP to solve this problem was reported in Xiao and Konak (2016), Matos et al. (2018). However, there is a non meta-heuristic method used to solve such the problem at the beginning. In 1964, Clarke and Wright performed a systematic way to solve such the problem by coining a "Clarke and Wright algorithm". The main feature of the algorithm is arranging some particular point-to-point paths started from the path with largest saving distance. The method has been improved recently: combination with tournament and roulette wheel selections (Pichpibul and Kawtummachai, 2013) and the combination with MILP (Li et al, 2016).

3. THEORETICAL BACKGROUND

3.1. Capacitated Vehicle Routing Problem (CVRP)

Capacitated Vehicle Routing Problem (CVRP) is a particular class of VRP considering a limited capacity (of load) of each vehicle. The objective is to find a least-cost path such that the requirement of the visited nodes can be satisfied. The scenario can be described as follows. Suppose that there are a set of vehicles with their own load capacity. The vehicles have to deliver loads from a centralized depot to some nodes and if there is still some unaccomplished requests from some particular nodes, then the vehicles have to come back to the depot and deliver some loads to the nodes. Finally, the vehicles have to return to the depot.

3.2. Clarke-Wright Saving Algorithm

Clarke Wright (CW) savings algorithm is a method proposed by Clarke and Wright (1964). This method intends to find a route with largest distance saving by a route exchanging mechanism. Originally, the problems handled by the algorithm are those without considering vehicle's capacity. However, as the vehicle routing problems become more complex, the algorithm was also combined to some meta-heuristics methods (Li et al, 2015; Li et al, 2016).

The principle of CW saving algorithm is to find the route such that the accumulation of saving distance can be maximized. Saving distance from a pair of the *i*-th and the *j*-th nodes ($S_{i,j}$) is defined as the difference of the distance between the depot-to-the *i*-th node and the depot-to-the *j*-node:

$$S_{i,j} = d_{D,i} + d_{D,j} - d_{i,j} \quad (1)$$

The algorithm can basically be described in the following steps. A distance matrix consisting the distance between any nodes in the graph then is constructed following by a saving matrix distance. The original algorithm is described in Algorithm 1.

Algorithm 1. Original Clarke-Wright Saving Algorithm.

1: Initialize the distance data, number of requests data, service time data, average vehicle speed and vehicle capacity as input needed.
2: Create a *distance matrix* between depots to consumers and between consumers to consumers.
3: Perform a saving matrix by using equation (1).
4: Perform a list of saving value calculated in step 3 from the largest to the smallest.
5: Determine the array of outlet assigned to the tour by selecting the outlet combination with the largest saving value.

4. METHODS

4.1. Problem Definition

Suppose that there exists a depot *D* and *N* outlets to serve. Let v_i represents the *i*-th node and $e_{i,j}$ represents the route connecting the *i*-th and the *j*-th nodes. The node indexed with 0 is dedicated to be a centralized depot. Moreover, let us define R_i as the requested amount of goods. Suppose that there are *K* non-homogeneous vehicles, and C_k, $k = 1, \ldots, K$ represent the capacity of the *k*-th vehicle. Also, suppose that in a day, there are *h* shifts given to all the vehicles to deliver the goods. Let us define T_{limit}, *h* as the maximum traveling time for all vehicles at the *h*-th session. Then we formulate the problem in a quadratic form as follows.

Min

$$J = \lambda_{\text{time}}\left(\sum_{h=1}^{H} \max_{k}\left\{t_{k,h}\right\}\right)^2 + \lambda_{\text{dist}}\left(\sum_{i=1}^{M}\sum_{j=1}^{M} a_{i,j} S_{i,j}\right)^{-2} \quad (2)$$

subject to

$t_{k,h} < T_{limit}$; for all $k = 1, \ldots, K$; $h = 1, \ldots, H$,

$0 \le c_k \le C_k$, for all $k = 1, \ldots, K$

where $t_{k,h}$ is the traveling time of the *k*-th vehicle in the *h*-th session; c_k is the load of the *k*-th vehicle; $S_{i,j}$ is the saving distance between the *i*-th and the *j*-th outlets; $a_{i,j}$ is the constant which is 1 if the vehicle travel from the *i*-th outlet to the *j*-th outlet and 0 otherwise; λ_{time} and λ_{dist} are time-related and distance constants, respectively. The quadratic form is selected under the consideration that this form always have global minimum.

Note that λ_{time} and λ_{dist} are used to determine the dominance level of time and saving distance to the function *J*, respectively. For instance, if λ_{time} is set very low and λ_{dist} is set very high, it possible that the saving distance is dominant. Consequently, the objective function *J* depends more on saving distance. The component of $t_{k,h}$ is considered because in actual situation the route between any two outlets (or from depot to outlet) may have traffic jam.

4.2. Modified Clarke-Wright Saving Algorithm

In this paper, we modify the traditional Clarke-Wright Saving (CW) algorithm by including the factors of delivery sessions and vehicle's capacity. Here, the saving matrix is not only performed by the saving distance $S_{i,j}$ but also the traveling time. Let us $\mathbf{D} = \{d_{i,j}\}$ as the matrix of outlet-to-outlet distance (denoted as $d_{i,j}$); $\mathbf{T} = \{t_{i,j}\}$ as the matrix of outlet-to-outlet traveling time (denoted as

$t_{i,j}$); **C** = {c_i} as the matrix of maximum demand of the *i*-th outlet.

The proposed modified CW algorithm has a main feature, i.e., the typical saving matrix is now improved by adding travelling time. Since the saving distance has to be maximized and the traveling time has to be minimized, therefore we design a cost function such that the distance and time components must be minimized.

The algorithm is described as follows.

Algorithm 1. Modified Clarke-Wright Saving Algorithm.

1 : Prepare **D, T,** and **C**.
2 : Prepare the list of route, denoted as $\mathbf{R}_{\text{solution}}$ = {$r_{i,j}$}, where $r_{i,j}$ is the route from the *i*-th to *j*-th outlets.
3 : **For** each pair of outlets in **D**,
4 : Calculate $J_{i,j}$ which is formulated in equation (2). Put the value of $J_{i,j}$ to the (*i*,*j*)-th element of a modified Clarke-Wright saving matrix, denoted as **M**.
5 : **End For**
6 : Make a list of outlet-to-outlet routes {$r_{i,j}$} from the smallest to the largest $J_{i,j}$, denoted as $\mathbf{R}_{\text{sorted}}$.

Algorithm 2. Split Delivery Mechanism

1 : Prepare $\mathbf{R}_{\text{sorted}}$.
2 : Prepare matrix of worst case capacity order, denoted by **O**.
3 : **For** all shifts, do the following steps:
4 : Select assigned vehicle from the vehicle pool, starting from the available largest capacity one.
5 : **For** each selected vehicle, do the following steps:
6 : Pick an available $r_{i,j}$ in $\mathbf{R}_{\text{sorted}}$ with smallest J from the outlet-to-outlet route list.
7 : Calculate load accumulation if $r_{i,j}$ is included in the solution.
8 : Calculate the remaining space available in the selected vehicle if $r_{i,j}$ is included in the solution.
9 : **If** the remaining load space in step 8 is less than the selected vehicle', then put $r_{i,j}$ in the solution. After that, go to step 6 for next route selection.
10 : **Else**,
11 : **If** there is a remaining demand in **C**,
12 : **If** there is at least one available vehicle,
13 : Go to step 4.
14 : **Else**
15 : Go to step 27.
16 : **End**
17 : **Else**
18 : **If** the traveling time exceeds the allowable time in the shift
19 : Remove the selected $r_{i,j}$
20 : Go to step 6.
21 : **Else**
22 : Go to step 27.
23 : **End**
24 : **End**
25 : **End**
26 : **End**
27 : Perform route combination.
28 : **If** the available shift exists
29 : Prepare for a new route.
30 : Goto step 3.
31 : **End.**
32 : **End.**

The case used in this study is a rice distributor located in Klaten, Central Java, Indonesia. The distributor has 9 outlets to serve, which are N1, …, N9. All the outlets are located in Yogyakarta, Indonesia. Each outlet commits a daily order to the distributor. We gathered 6-month order data from each outlet and we categorize them into daily orders. The maximum daily order in each outlet is presented in Table 1.

As a preliminary step for calculating the optimal routes, we applied a survey to gather data for **D** (depot-to-outlet and outlet-to-outlet distances), **T** (outlet-to-outlet traveling time), and **C** (maximum demand), which are showed in Tables 1, 2, and 3, respectively. The vehicles used are of two categories. The first category is the vehicle with capacity of 2500 kg and the second one is of 1300 kg. According to Algorithm 2, the first vehicle is the priority to be full-loaded.

5. RESULTS AND DISCUSSION

The modified Clarke-Wright algorithm proposed in Algorithms 1 and 2 has a feature of combining time and 1/saving distance. Therefore, we applied the cost function (2) to the original CW saving matrix. The resulted routes for each day are shown in Table 5. The first impression from the result is that there is no undelivered load, since the provided vehicles and shifts are appropriate enough. The effect of traffic jam (or, the observed travelling jam) appears in the resulted routes. It can be investigated that the route is influenced mostly by the demand. As an example, the route for V1 in Shift 1 starts from the depot (symbolized by Dt) and continues to N9, N6, and N8. Different with the results in Monday, Thursday, Friday, and Saturday, the routes for the vehicle V1 in Shift 1 in Tuesday and Wednesday exclude N5. As shown in Table 3, in Tuesday and Wednesday, the demand from N9, N6, and N8 are high. In Tuesday, the total demand of N9, N6, and N8 is 2400 kg, which can be delivered by the first vehicle (V1) in Shift 1. On the other hand, the demand of four outlets in Monday, i.e., N9, N6, N8, and N5, can be accomplished by the vehicle 1 in Shift 1 because of the total demand is 2150 kg. The remaining space is 350 kg. However, this space cannot be fulfilled since there is no demand that is less than 350 kg from any other outlet in that day.

Another pattern in this result is that the decision is dominated by the traveling time. From Table 1, we can conclude that the reciprocal of the saving distance is significantly small compared to traveling time in Table 2. This can be proofed by the multi-objective matrix in Table 4, where the outlet-to-outlet objective values are closed to the traveling time in Table 2.

Table 1. Outlet-to-outlet distance matrix

From/To	Depot	N1	N2	N3	N4	N5	N6	N7	N8	N9
Depot	-									
N1	43.16	-								
N2	45.41	6.86	-							
N3	44.61	6.04	4.01	-						
N4	45.16	5.81	4.51	0.77	-					
N5	45.90	4.99	6.40	4.37	3.39	-				
N6	46.16	6.97	5.31	1.84	2.47	4.92	-			
N7	45.04	4.68	7.40	3.66	4.39	2.54	2.81	-		
N8	48.8	11.4	5.87	5.15	7.80	9.05	6.28	7.21	-	
N9	44.61	3.66	4.22	2.10	1.12	2.49	8.01	3.13	8.77	-

Table 2. Longest traveling time (in minutes)

From/To	Depot	N1	N2	N3	N4	N5	N6	N7	N8	N9
Depot	-									
N1	106.43	-								
N2	109.36	11.38	-							
N3	108.47	15.23	10.4	-						
N4	109.17	13.21	11.54	6.16	-					
N5	111.18	13.57	15.3	13.58	14.57	-				
N6	108.39	22.14	12.22	6.48	6.29	16.33	-			
N7	113.5	10.26	15.28	11.59	12.28	6.29	7.5	-		
N8	123.24	25.49	21.22	12.36	23.45	28.47	32.31	23.22	-	
N9	110.24	11.2	12.03	7.22	6.44	14.22	33.42	7.31	23.1	-

Table 3. Maximum order of each outlet in 6 month

Outlets	Daily highest weight (kg)					
	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
N1	375	360	310	245	275	400
N2	650	700	800	700	800	800
N3	375	500	800	800	600	225
N4	800	975	725	550	650	725
N5	375	350	325	350	375	325
N6	600	750	675	500	750	400
N7	600	650	650	625	350	625
N8	275	775	700	625	300	325
N9	900	875	945	800	825	850
Total demand	4950	5935	5930	5195	4925	4675

Tabel 4. Multi-objective matrix

From/To	N1	N2	N3	N4	N5	N6	N7	N8	N9
N1	-								
N2	129.505	-							
N3	231.953	108.160	-						
N4	174.504	133.172	37.946	-					
N5	184.145	234.090	184.417	212.285	-				
N6	490.180	149.329	41.991	39.564	266.669	-			
N7	105.268	233.479	134.328	150.799	39.943	56.250	-		
N8	649.740	450.289	152.770	549.903	810.541	1043.936	539.169	-	
N9	125.440	144.721	52.129	41.474	202.209	1116.897	53.436	533.610	-

Table 5. Resulted distribution routes

Day	Shift	Vehicles	Route	Total Load (kg)	Total Time (min)	Total Distance (km)
Monday	Shift 1	T1	Dt-N9-N6-N8-N5-Dt	2150	357.25	108.88
		T2	Dt-N7-N2-Dt	1250	264.69	97.85
	Shift 2	T1	Dt-N3-N1-N4-Dt	1550	288.10	101.62
Tuesday	Shift 1	T1	Dt-N9-N6-N8-Dt	2400	337.23	103.54
		T2	Dt-N5-N2-Dt	1050	263.25	96.9
	Shift 2	T1	Dt-N3-N1-N4-N7-Dt	2485	310.32	105.89
Wednesday	Shift 1	T1	Dt-N9-N6-N8-Dt	2320	327.93	103.54
		T2	Dt-N5-N2-Dt	1125	258.06	96.9
	Shift 2	T1	Dt-N3-N1-N4-N7-Dt	2485	311.18	105.89

Day	Shift	Vehicles	Route	Total Load (kg)	Total Time (min)	Total Distance (km)
Thursday	Shift 1	T1	Dt-N9-N6-N8-N5-Dt	2275	356.98	108.88
		T2	Dt-N3-N1-Dt	1045	259.83	93.81
	Shift 2	T1	Dt-N7-N2-N4-Dt	1875	292.72	102.11
Friday	Shift 1	T1	Dt-N9-N6-N8-N5-Dt	2250	358.78	108.88
		T2	Dt-N7-N2-Dt	1150	268.28	97.85
	Shift 2	T1	Dt-N3-N1-N4-Depot	1525	284.73	101.62
Saturday	Shift 1	T1	Dt-N9-N6-N8-N5-Dt	1900	359.47	108.88
		T2	Dt-N7-N3-N1-Dt	1250	282.69	97.9
	Shift 2	T1	Dt-N4-N2-Dt	1525	258.81	94.59

6. CONCLUSIONS

In this paper, a modified Clarke-Wright algorithm for generating routes is proposed. The main feature is the addition of outlet-to-outlet traveling time in the typical Clarke-Wright matrix. The traveling time is included in the cost function as the consequence of the presence of traffic jam in the routes. A case of a rice distributor with nine outlets is taken. In this case, two vehicles with limited capacities that can operate in two shifts are considered. As the result, a set of routes that are assigned to the available vehicles are generated. There are two patterns that can be observed from the results, i.e., the routes are highly influenced by the amount of demand from the outlets, and also by the outlet-to-outlet traveling time.

Future research of this study will be focused on some issues, i.e., the elimination of priority in the selection of vehicles, the consideration of traveling expenses, and the application of vehicle's dynamic constraints.

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