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An Inventory Model to Control Multi-Item Medicines with Consideration of Space Capacity in the Hospital and Joint Replenishment

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ABSTRACT

Medicines can be considered the most widely used medical expenditures in hospitals. To reduce the medicines expenses in the hospital, the term of inventory control is applied. Medicines must be controlled by considering the expiry dates and the probabilistic demand from the customers. Therefore, it is necessary to develop an inventory model that can be suitable to control medicines by minimizing the expired medicines, total inventory costs, and dealing with unpredictable demand. The purpose of this research is to develop an inventory model for determining optimum replenishment time and order quantities and space requirements for multi-item medicines with consideration of expiry dates of the medicines and all medicines are being purchased in a single purchase order so that the total inventory costs in hospitals can be minimized. The result is that the proposed inventory model results in optimum space requirements and the lowest inventory costs. Therefore, hospitals must order medicines based on the optimum order quantity.

Keywords: multi-item medicines, expired medicines, joint replenishment, space requirements.

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1. INTRODUCTION

Pharmaceuticals or medicines are considered one of the most widely used medical facilities in the hospital (Palanisamy and Ranganathan, 2016). Kelle et al. (2012) stated that pharmaceuticals or medicines would spend around 10% of a hospital or medical expenditures in a hospital. Research states that 10-18% of total revenues are being spent on inventory costs, and those materials are characterized as a large number of operating expenses (Balasubramanian and Nivetheni, 2018). Besides dealing with inventory costs, hospitals must also reduce the number of expired medicines in storage. Expired medicines are caused by an excessive amount of medicines in storage. Expired medicines cannot be sold, and it will cause profit loss to the hospital.

In order to reduce the number of expired medicines and inventory costs, it is necessary for the hospital to adopt the term of inventory control. The purpose of applying the inventory control method is to determine the optimum order quantity for the hospitals. There will be either no lack of medicines or excessive medicines in the hospital. Lack of pharmaceuticals or medicines gives more losses than the losses of general products (Li et al., 2018), and excessive medicines will cause expired medicines. Both lack and excessive medicines will cause the hospitals to suffer more inventory costs or operating expenses.

Medicines can be considered as perishable items. Almarsdóttir and Traulsen (2005) stated that there are differences of fundamentals between controlling medicines and other perishable items. Then, Li et al. (2018) proposed that controlling medicines or pharmaceuticals must be considered about the expired date, and it needs special consideration on the inventory control. Therefore, the inventory model must be developed to be suitable for being applied in controlling medicines.

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Hospitals always face uncertain demand in controlling the number of medicines. Uncertainty will lead them to uncertain order quantity to keep the stock of medicines in the storage. If the medicines are ordered either in too few or large amounts, it will give impacts which have been mentioned above. To deal with uncertain demand, and inventory model must be developed with consideration of probabilistic demand. Besides uncertain demand, a hospital must have limited capacity to store the medicines. If medicines are ordered at an excessive level, then there will be no space to store the medicines, which leads to medicines' expired medicines and expires medicines will lead to profit loss to hospitals because they cannot be sold again to the end customers (patients).

Based on the problems above, the purpose of this research is to develop an inventory model to control multi-item medicines with consideration of probabilistic demand and shelf-life (or expiry date of the medicines). The inventory model also considers determining the optimum order quantity so that the quantity will not exceed the space capacity in the hospital. Because hospitals often order certain medicines from a single supplier to minimize ordering costs, the inventory model will also consider the term of purchasing in a single purchase order (or joint replenishment). In short, the objective of this paper is too controlling probabilistic multi-item medicines in a single purchase order with consideration of expiry dates of the medicines and space capacity in the storage.

2. LITERATURE REVIEW

2.1. Inventory in Hospitals

Inventory is one of the most important assets in a hospital. Types of inventory in hospitals are supplies, drugs (or medicines), and equipment stored in the hospitals. Supplies in hospitals include single-use items (such as needles, syringes, etc.), items that are used in short period or expendables (such as tape, cotton wool, etc.), reusable items (such as catheters and sterilizable syringes), and items with short life span (such as thermometers). Equipment in a hospital is items that can be used or durable for an extended period of time (such as beds, microscopes, sterilizers, weighing scales, stethoscopes, etc.).

2.2. Factors that Affect Inventory in Hospitals

According to West inside the journal from Dwidevi et al. (2012), types of products, sizes of the inventory, unclaimed prescriptions, inventory shrinkage (such as robbery activities), returned product policies, and use of formularies are the following factors that affect the inventory in pharmaceuticals and hospitals. Based on the research conducted by Onkundi and Bichanga (2016), there are several factors that influence the performance of inventory: stock replenishment (duration of replenishment), information sharing, and inventory costs, and demand variability. Then, types of demand (deterministic and probabilistic demand) can also become a major factor that influences the performance of inventory.

2.3. Inventory Model based on Demand

Based on the type of demand, the inventory model can be classified into two categories. Those are the deterministic inventory model and probabilistic inventory model.

If the elements of the inventory model (demand, lead time, and order quantity) can be precisely determined, that model can be considered a deterministic inventory model and vice versa. Deterministic demand can be solved by applying the EOQ (Economic Order Quantity) model to determine optimum order quantity, reorder point, and safety stocks.

If the elements of the inventory model cannot be precisely determined, that model can be considered a probabilistic inventory model. The probabilistic EOQ model can be applied to solve probabilistic demand by considering types of demand rate, partial backlogging, and time holding cost (Agada and Ogwuche, 2017).

2.4. Inventory Model based on Supply Chain Perspectives

Besides demand, the inventory model can be classified into two types based on supply chain perspectives. Those are single items and multi-item. According to Rusli et al. (2014), the single item inventory model is an inventory model which is used to order one type of raw materials or items or products from a single supplier. The multi-item inventory model is an inventory model which is used to order more than one or several types of raw materials or items or products from a single supplier.

The mathematical model for controlling single items and multi-item is also different. Then, to determine the total inventory cost for a single item, the basic equation is the sum of purchasing, ordering, and holding costs. From a multi-item perspective, it is not necessary to sum all the inventory costs as stated in the single item perspective. However, the first equation is to calculate the interval ordering period.

2.5. Contribution for this Paper

Contribution for this paper can be conducted by comparing previous papers with recent papers.

To control medicines in hospitals, there are several inventory control models that can be applied. One of the models is classifying medicines into several categories, and the most commonly used classification method is ABC (Always Better Control) Analysis. Dwivedi et al. (2012) also added that it is feasible and efficient to apply ABC Analysis because this model is used for facing uncertain demand conditions, which occur mostly in hospitals. Another research that has been conducted by Ramanathan (2006) also stated that ABC Analysis could classify or identify medicines that need greater attention of control. This analysis is being determined as a worldwide method to control medicines in hospitals. In other research, ABC Analysis is combined with another categorization model in which that model is called "bi-criteria". In "bi criteria", the analysis can be performed by categorizing medicines into two different categories, and the joint – matrix will be performed in applying the "bi-criteria".

Medicines can be controlled by using analytical methods. One of the most common analytical methods is EOQ (Economic Order Quantity) model. EOQ concerns static demand, and the function of EOQ is used to calculate optimum order quantity for the medicines. Nevertheless, demand is usually uncertain in real-life conditions, so that the EOQ model must be developed so that it can be applied in real-life conditions. Therefore, the term of probabilities EOQ model is being applied. In addition, Agada and Ogwuche (2017) also stated that stochastic demands with high variations always occur in real-life conditions. Besides controlling medicines by using the EOQ model, medicines can be controlled by considering the review system. Some research applied periodic review to control medicines in hospitals, and others applied continuous review to control medicines. Medicines that are being controlled by applying a continuous review period will lead to the application of order quantity and reorder point.

Medicines must not be ordered at an excessive level because hospitals have limited capacities to store the medicines. So, the model must be developed so that the order quantity of the medicines must not exceed the capacities of the hospitals. To deal with capacities constraints, Kelle et al. (2012) developed a multi-item inventory model. The purpose of the model is to determine the optimum order quantity with the limitation of space.

Medicines can be considered as perishable items because every medicine will have its expiry date. Therefore, medicines cannot be controlled in the same way as controlling deteriorating products (Uthayakumar and Priyan, 2013). Special considerations need to be required on controlling those two materials. The fundamentals are different between medicines or pharmaceuticals and other perishable products (Almarsdóttir and Traulsen, 2005). Besides that, controlling medicines or pharmaceuticals must be considered about the expired date, and it needs special consideration on the inventory control (Li et al., 2018). The reason is that shelf life will have a major impact on the performance of inventory management in the hospital because it will cause profit loss and stock fails in the hospitals' storage. If a hospital order excessive quantity of medicines, then there will be leftover medicines in the storage which lead the medicines to become expired when the medicines reach the expiry date. If the medicines are ordered in small quantities, it will cause stockout conditions so that demands from customers will not be satisfied. Therefore, the inventory model must be developed so that it can be suitable to control medicines in hospitals. To deal with expired medicines. Puspitasari et al. (2016) developed an inventory model to control single-item medicines with probabilistic demand. Then, Li et al. (2018) developed an inventory model with consideration of shelf life and the stochastic demand

There is much research that has developed an inventory model to control medicines in several conditions. Therefore, this research will focus on developing an inventory model for controlling medicines based on expiry dates with consideration of optimal order quantity for each medicine. In addition, this research will consider developing a multi-item inventory model considering expiry date, probabilistic demand, orders in a single purchase order, and space availability in the storage. Mostly one supplier supplies more than one type of medicine (multi-item from supply chain perspectives). So, joint order (or joint replenishment) in a single purchase order will also become the main focus of developing the inventory model.

Because demands are not constant in real-life conditions, then the type of demands of medicines in this research is assumed to be probabilistic. In addition, each hospital must have a limited capacity to store the medicines. Suppose medicines are ordered at an excessive level. In that case, there will be no space to store the medicines, which leads to medicines' defective medicines, and defective medicines will lead to profit loss to hospitals because they cannot be sold again to the end customers (patients) and vice versa. If medicines are ordered at a small level, there will probably be a lack of medicines, which leads to patients' dissatisfaction, and demands from patients cannot be fulfilled in that time. In conclusion, this research has an objective to controlling probabilistic multi-item medicines in a single purchase order (or joint replenishment or joint order) with consideration of expiry dates of the medicines and space capacity in the storage.

3. METHODOLOGY

The model development begins with setting assumptions, and the assumptions are listed as follows.

a. Lead time is fixed for all medicines (Agada and Ogwuche, 2017)
b. All items have the same lead time and expiry date (Puspitasari et al., 2016)
c. There will be shortage conditions (Puspitasari et al., 2016)
d. Demand distribution is assumed to be uniformly distributed (El-Wakeel, 2011)
e. All medicines are purchased in a single supplier in one purchase order
f. Medicines are assumed to be multi-item from the supply chain perspective (Kelle et al., 2012)
g. The prices of ordered goods remain the same and do not depend on the order quantity of the medicines (Pratiwi et al., 2018)
h. Expiry period or shelf life for all medicines are known (Li et al., 2018)
i. Ordering cost does not depend on the order quantity of the medicines (Pratiwi et al., 2018)
j. Medicines of a single supplier will be stored in the same storage
k. A continuous review system (Q,r) is being applied in this research
l. No historical data on medicines' assumptions are being recorded
m. Service levels for each medicine are being assumed

After setting the assumptions, next is to determine the notations that are going to be used. The notations are listed as follows.

K_i : setup or order cost per cycle

a_i : lower bound of consumption rate (uniform) for medicine i, i = 1,2,3,… n

b_i : upper bound of consumption rate (uniform) for medicine i, i = 1,2,3,… n

D_i : average demand per year for medicine i, i = 1,2,3,…n

Q_i : optimal order quantity for medicine i, i = 1,2,3,…n

Q_{e_i}: expired quantity for medicine i, i = 1,2,3,…n

r_i : reorder point for medicine i, i = 1,2,3,… n

r_i - $E[X]_i = (SS)_i$: safety stock or expected net inventory for medicine i, i = 1,2,3,…n

$E[X]_i$: expected demnd for medicine i, i= 1,…,n

h_i : holding cost for item i

p_i : purchasing cost for item i

B_i : shortage cost for item i

z_i : service level for item i

f_i : space required or volume for item i

w : optimum total space required in the volume unit

y : determined space required in the volume unit

t_{1_i} : time between item i arrival to expired

t_{2_i}: time during medicines in storage is zero or medicines are expired

t_L: time between medicines are ordered until the medicines arrive

m: medicines shelf life (year)

s_i : lot size per box for item i

T^* : optimum replenishment time or ordering length

The reason for using uniform distribution is that there are no historical demand data for the medicines. So, the demand probability density function will be unknown. To overcome this situation, Wanke (2008) concluded that uniform distribution is reasonable when determining the lot size and reorder point for unknown distribution demand.

There are six steps to generate the new inventory model based on the purpose of this paper. The steps will be explained in the sections below.

3.1. Basic Inventory Costs in Multi-Item Perspectives

Kelle et al. (2012) and Agada and Ogwuche (2017) stated that there are three costs in order to calculate the total inventory costs in a period of time, and those costs are represented in the equation below.

$$Min\ TIC_i = TOC_i + E(THC)_i + E(TBC)_i \quad (1)$$

where TOC_i is the total ordering costs, $E(THC)_i$ is the expected holding costs, and $E(TBC)_i$ is the expected shortage costs.

The equation for calculating TOC_i is:

$$TOC_i = K_i \frac{D_i}{Q_i} \quad (2)$$

Total expected holding costs can be calculated by considering inventory level at the beginning of the period and inventory level at the end of a cycle. The equation for inventory level at the beginning of the period is:

$$Beginning\ Inventory = r_i - E[X]_i + Q_i \quad (3)$$

The equation for inventory level at the end of the cycle is:

$$Ending\ Inventory = r_i - E[X]_i = (SS)_i \quad (4)$$

Because it is uniformly distributed, then the equation of $E[X]_i$ is:

$$E[X]_i = \frac{b_i - a_i}{2} \quad (5)$$

The equation for calculating reorder point is:

$$r_i = \left(z_i \times (b_i - a_i)\right) + a_i \quad (6)$$

So, the equation for the expected inventory level can be calculated by adding both the equation of inventory level at the beginning of the period and the inventory level at the end of the cycle. So, the equation can be written as:

$$E[X]_i = \frac{1}{2}(r_i - E[X]_i + Q_i + r_i - E[X]_i)$$

$$E[X]_i = \frac{Q_i}{2} + r_i - E[X]_i = \frac{Q_i}{2} + r_i - (\frac{b_i - a_i}{2}) \quad (7)$$

Therefore, the equation for calculating the total expected holding costs $E(THC)_i$ is :

$$E(THC)_i = h_i p_i \left(\frac{Q_i}{2} + r_i - E[X]_i\right) = h_i p_i \left(\frac{Q_i}{2} + r_i - (\frac{b_i - a_i}{2})\right) \quad (8)$$

Agada and Ogwuche (2017) stated that shortages occur when the condition is $X>R$, where X is demand, and R is inventory level. A shortage occurs when stocks in the storage cannot fulfill the demand from the customers. Figure 1 shows the inventory model with shortages.

Then, the expected shortage in a cycle is:

$$N = (B(r))_i = \int_{-\infty}^{\infty} (X_i - r_i) f_i(X_i)\, dX_i$$

$$(B(r))_i = \int_{r_i}^{\infty} (X_i - r_i) f_i(X_i)\, dX_i \quad (9)$$

By using the equation of $f_i(x_i) = \frac{1}{b_i - a_i}$, and $\int_{r_i}^{\infty} f_i(x_i) d(x_i) = 1 - \frac{r_i}{b_i - a_i}$ in uniform distribution and inserting to equation (5), then the expected shortage is :

$$N_i = (B(r))_i = \int_{r_i}^{\infty} (X_i) \frac{1}{b_i - a_i} d(x_i) - r_i \int_{r_i}^{\infty} f_i(x_i) d(x_i)$$

$$N_i = (B(r))_i = \frac{r_i^2}{2(b_i - a_i)} + \frac{(b_i - a_i)}{2} - r_i \quad (10)$$

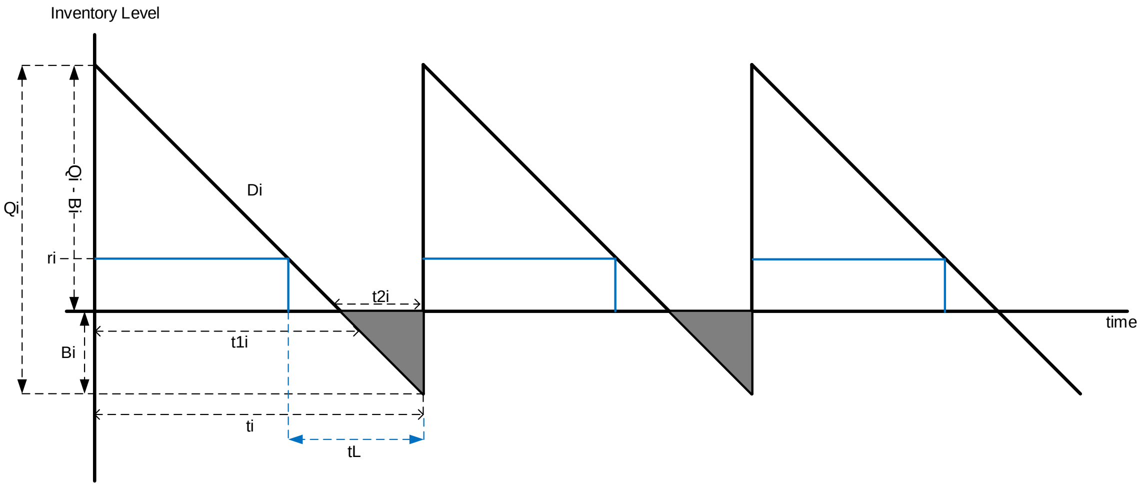

Figure 1. Inventory Model with Shortage

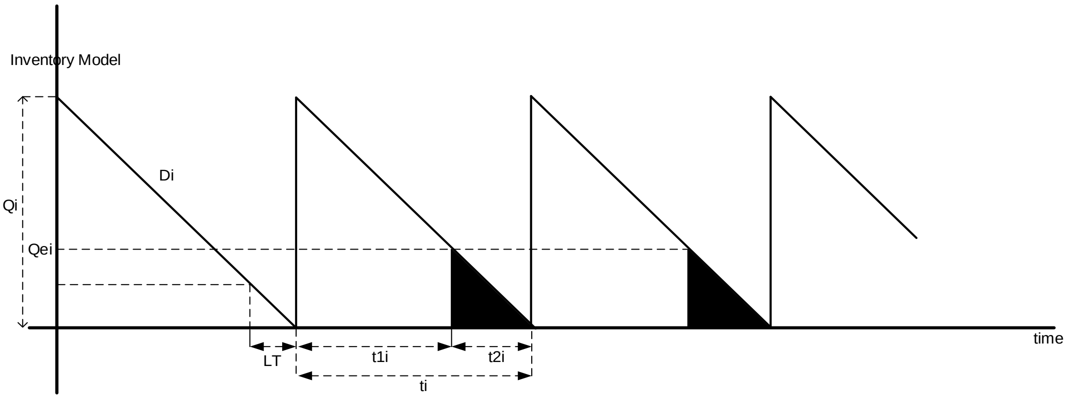

Figure 2. Inventory Model with Expiry Date

3.2. Inventory Costs to Control Multi-Item Medicines with Expiry Date

The basic equation to control multi-item medicines adopts the model that has been developed by Puspitasari et al. (2016) and Amran and Fatima (2017). Figure 2 shows the inventory model with different purchase orders.

The equation which has been developed by Amran and Fatima (2017) stated that there would be no expired quantities when $m > t_i$ and vice versa. When the condition is $m \leq t_i$, then expired quantities will occur. When $m > t_i$, then the equation of total inventory costs will be the same as the basic total inventory costs. But, when the condition is $m \leq t_i$, then expired costs will be added to the total inventory costs.

Expired medicines must be destroyed so that expired medicines will not be stored and calculated in the holding costs. Then, the equation for average inventory level will be multiplied with the number of cycles per year, and the equation will become:

$$E[Inventory\ Level]_i=\left[\left(\left(\frac{Q_i-Q_{e_i}}{2}\right)\times t_{1_i}\right)\times\frac{D_i}{Q_i}\right]+(SS)_i \quad (11)$$

By using the congruence principle, then the equation of t_{1_i} is:

$$t_{1_i}=\frac{Q_i-Q_{e_i}}{D_i} \quad (12)$$

By substituting equation (12) to equation (11), then the equation for expected holding cost is:

$$E[Inventory\ Level]_i=\frac{\left(Q_i-Q_{e_i}\right)^2}{2Q_i}+(SS)_i$$

$$E(THC)_i=h_i\left[\frac{\left(Q_i-Q_{e_i}\right)^2}{2Q_i}+(SS)_i\right] \quad (13)$$

NB: the equation of $(SS)_i$ is the same equation with equation (4), and the equation is going to be substituted when calculations are going to be conducted.

Shortage because of expired products occurs when the level of inventory reaches the period t_{2_i} and the equation of t_{2_i} is:

$$t_i=t_{1_i}+t_{2_i}$$

$$t_{2_i}=\frac{Q_{e_i}}{D_i} \quad (14)$$

The inventory life cycle ($E[X]_i$) during period t_{2_i} is:

$$E[X]_i=\frac{Q_{e_i}}{2}\times t_{2_i}$$

$$E[X]_i = \frac{Q_{e_i}}{2} \times \frac{Q_{e_i}}{D_i} = \frac{\left(Q_{e_i}\right)^2}{2D_i} \tag{15}$$

Then, the equation for calculating the expected shortage cost because of unsatisfied demand and the expired product is:

$$E(TBC)_i = \frac{B_i D_i}{Q_i}\left[(N_i) + \left(\frac{\left(Q_{e_i}\right)^2}{2D_i}\right)\right] \tag{16}$$

NB: the equation of N_i is the same equation with equation (10), and the equation is going to be substituted when calculations are going to be conducted.

Expired cost occurs when there are some medicines that have been expired and cannot be returned to the supplier. The inventory life cycle ($E[X]_i$) during period t_{2_i} is the same with equation (15). Therefore, the equation for calculating the total expired cost is:

$$TEC_i = p_i \frac{D_i}{Q_i} \times \left(\frac{\left(Q_{e_i}\right)^2}{2D_i}\right)$$

$$TEC_i = p_i \times \frac{\left(Q_{e_i}\right)^2}{2Q_i} \tag{17}$$

The equation of ordering costs remains the same with equation (2). Then, the inventory costs for controlling multi-item medicines with expiry dates can be generated. Hence, the total inventory costs are:

$$TIC_i = K_i \frac{D_i}{Q_i} + h_i\left[\frac{\left(Q_i - Q_{e_i}\right)^2}{2Q_i} + (SS)_i\right] + \left[\frac{B_i D_i}{Q_i}\left[(N_i) + \left(\frac{\left(Q_{e_i}\right)^2}{2D_i}\right)\right]\right] + \left[p_i \times \frac{\left(Q_{e_i}\right)^2}{2Q_i}\right] \tag{18}$$

By setting derivative equals to zero, then the equation for expired quantity (Q_{e_i}) is:

$$Q_{e_i} = \frac{h_i Q_i}{(h_i + B_i)} \tag{19}$$

3.3. Inventory Costs to Control Multi-Item Medicines with Expiry Date and Joint Replenishment (Delivered in a Single Purchase Order)

This research will focus on controlling multi-item medicines in a single purchase order. Because all the items are being purchased in a purchase order, the ordering cycle must be the same for all items in a purchase order. Figure 3 shows the multi-item inventory model with a single purchase order.

In Figure 3, it shows that T^* is the lead time for ordering from one cycle to another cycle, and because it is in the multi-item perspective, so the ordering cycle for all items must be the same. Thus, the equation is:

$$T^* = \frac{D_i}{Q_i} = \frac{D_1}{Q_1} = \frac{D_2}{Q_2} = \ldots = \frac{D_n}{Q_n} \tag{20}$$

Therefore, the formula of finding Q_i is:

$$Q_i = T^* \times D_i \tag{21}$$

All costs which have been stated above must be substituted with equation (21). Therefore, the ordering cost related to a single purchase order is:

$$TOC = \frac{K}{T^*} \tag{22}$$

The total holding cost equation related to all items in a single purchase order is the sum of all holding costs of the medicines. Then, the inventory cycle t_{1_i} can be determined by using the congruence principle. By using the congruence principle, the equation of t_{1_i} is:

$$\frac{Q_i}{T^*} = \frac{Q_i - Q_{e_i}}{t_{1_i}}$$

$$t_{1_i} = \frac{T^*(Q_i - Q_{e_i})}{Q_i} = \frac{Q_i - Q_{e_i}}{D_i} \tag{23}$$

From equation (23), it can be seen that the inventory cycle of equation (12) is the same as equation (23). Therefore, the equation of holding costs from equation (13) with the sum of all holding costs for all medicines can be applied to determine the equation of total expected holding cost with a single purchase order. Then, the equation of total holding costs related to a single purchase order is:

$$E(THC) = \sum_{i=1}^{n} \frac{h_i\left(Q_i - Q_{e_i}\right)^2}{2Q_i} + h_i (SS)_i \tag{24}$$

$$E(THC) = T^* \sum_{i=1}^{n} \frac{[h_i D_i (h_i + B_i)((h_i + B_i) - h_i)] + h_i^3 D_i}{2(h_i + B_i)^2} + \sum_{i=1}^{n} h_i (SS)_i$$

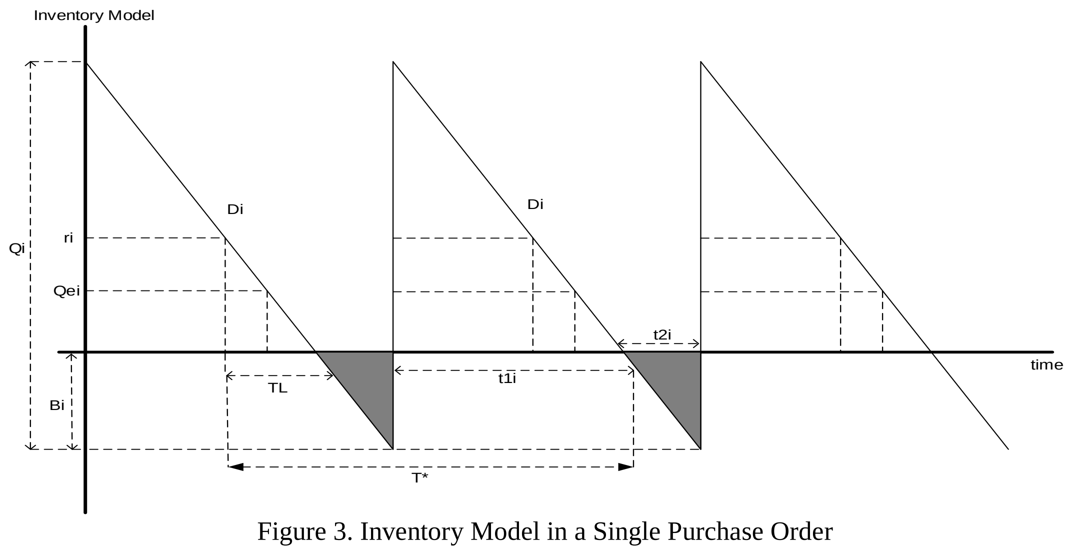

Figure 3. Inventory Model in a Single Purchase Order

From equation (23), it can be seen that the inventory cycle of equation (12) is the same as equation (23). Therefore, the equation of holding costs from equation (13) with the sum of all holding costs for all medicines can be applied to determine the equation of total expected holding cost with a single purchase order. Then, the equation of total holding costs related to a single purchase order is:

$$E(THC)=\sum_{i=1}^{n}\frac{h_i\left(Q_i-Q_{e_i}\right)^2}{2Q_i}+h_i(SS)_i \quad (24)$$

$$E(THC)=T^*\sum_{i=1}^{n}\frac{[h_iD_i(h_i+B_i)((h_i+B_i)-h_i)]+h_i^3D_i}{2(h_i+B_i)^2}+\sum_{i=1}^{n}h_i(SS)_i$$

The equation of total shortage cost related to all items in a single purchase order is the sum of all shortage costs of the medicines. Shortage condition occurs when inventory level reaches t_{2_i}. Therefore, the equation of t_{2_i} by using congruence principle is:

$$t_{2_i}=\frac{Q_{e_i}}{D_i} \quad (25)$$

Because equation (25) equals equation (14), then the equation for calculating the total expected shortage costs related to a single purchase order is the sum of total shortage costs from equation (16). Therefore, the equation of the total expected shortage costs related to a single purchase order is:

$$E(TBC)=\sum_{i=1}^{n}\frac{B_iD_i}{Q_i}\left[(N_i)+\left(\frac{\left(Q_{e_i}\right)^2}{2D_i}\right)\right]$$

$$E(TBC)=\sum_{i=1}^{n}\frac{B_i}{T^*}(N_i)+T^*\sum_{i=1}^{n}B_i\left(\frac{h_i^2D_i}{2(h_i+B_i)^2}\right) \quad (26)$$

Equation (17) can be applied to determine the equation of total expired costs related to the single purchase order. Therefore, the equation of the total expired costs related to a single purchase order is:

$$TEC=\sum_{i=1}^{n}p_i\left[\frac{\left(Q_{e_i}\right)^2}{2Q_i}\right]$$

$$TEC=T^*\sum_{i=1}^{n}p_iD_i\left[\frac{h_i^2}{2(h_i+B_i)^2}\right] \quad (27)$$

3.4. Determine the Value of Replenishment Time and Order Quantity

The total inventory costs controlling multi-item medicines in consideration of joint replenishment (single purchase order) and expiry dates are the sum of equation (22), equation (24), equation (26), and equation (27). By setting derivative equals zero, then the equation of replenishment time (T^*) is:

$$T^*=\sqrt{\frac{\left(K+\sum_{i=1}^{n}B_i(N)_i\right)}{\left[\sum_{i=1}^{n}\frac{[h_iD_i(h_i+B_i)((h_i+B_i)-h_i)]+h_i^3D_i}{2(h_i+B_i)^2}+B_i\left(\frac{h_i^2D_i}{2(h_i+B_i)^2}\right)+\sum_{i=1}^{n}\frac{p_ih_i^2D_i}{2(h_i+B_i)^2}\right]}} \quad (28)$$

By substituting equation (28) to equation (21), the optimum order quantity (Q_i) for each item is:

$$Q_i=D_i\times\sqrt{\frac{\left(K+\sum_{i=1}^{n}B_i(N)_i\right)}{\left[\sum_{i=1}^{n}\frac{[h_iD_i(h_i+B_i)((h_i+B_i)-h_i)]+h_i^3D_i}{2(h_i+B_i)^2}+B_i\left(\frac{h_i^2D_i}{2(h_i+B_i)^2}\right)+\sum_{i=1}^{n}\frac{p_ih_i^2D_i}{2(h_i+B_i)^2}\right]}} \quad (29)$$

Finally, the equation for calculating expired quantity (Q_{e_i}) can be calculated by substituting from equation (29) to equation (19).

3.5. Determine the Space Requirements for Multi-Item Medicines

After obtaining the optimum replenishment time (T^*) and order quantity (Q_i) for each medicine, then the space required for the multi-item medicines can be developed. Then, the size of space for storing the quantity (Q_i) with determined replenishment time (T^*) is:

$$w=\sum_{i=1}^{n}f_iQ_i \quad (30)$$

If $w > y$, then the order quantity for w_j for each medicine is less than order quantity for w for each medicine, but $TIC(w) < TIC(y)$ because there are any occurrences of shortage. If $w < y$, then the space required w will be implemented because if the order quantity for the medicines fulfill the space w_j, then holding cost will be higher, and there will be many occurrences of expired medicines. The effect is $TIC(w) < TIC(y)$.

Therefore, if the condition of $w < y$ occurs, then the condition will change to $y = w$, and the total cost will become $TIC(w) = TIC(y)$.

3.6. Model Validation

Equation (28) must be validated, and the equation (28) must also show the same model with the EOQ multi-item where the equation of EOQ multi-item is:

$$T^*=\sqrt{\frac{2K}{\sum_{i=1}^{n}h_iD_i}} \quad (31)$$

Because the equation of EOQ multi-item does not consider purchasing cost and backorder cost, then the shortage cost (B_i) and purchasing cost (p_i) will be zero. By setting $(SS)_i=N_i=p_i=u_i=0$, then the equation of T^* in developed model is :

$$T^*=\sqrt{\frac{\left(K+\sum_{i=1}^{n}0\right)}{\left[\sum_{i=1}^{n}\frac{[h_iD_i(h_i+0)((h_i+0)-h_i)]+h_i^3D_i}{2(h_i+0)^2}+B_i\left(\frac{h_i^2D_i}{2(h_i+0)^2}\right)+\sum_{i=1}^{n}\frac{p_ih_i^2D_i}{2(h_i+0)^2}\right]}}$$

$$T^*=\sqrt{\frac{2K}{\left[\sum_{i=1}^{n}\frac{h_i^3D_i}{(h_i)^2}\right]}}=\sqrt{\frac{2K}{\sum_{i=1}^{n}h_iD_i}}$$

The computed equation shows that the developed model is equal to the basic EOQ multi item model where Equation (28) equals Equation (31). It means that the model is valid to be used in real-life conditions to control medicines in a single purchase order with consideration of expiry dates and space requirements.

4. NUMERICAL EXAMPLE

4.1. Model Implementation

The numerical illustrations that are going to be implemented in this research are based on the assumption. Table 1 shows the parameter for numerical illustrations.

Table 1. Data and Parameter for Numerical Illustrations

Parameters	Medicines Type		
	A	B	C
D_i (in stripes)	0 – 100	0 – 110	0 – 90
f_i (in m^3)	0.3	0.25	0.2
s_i (stripes)	10	10	10
p_i (in IDR)	65,000	70,000	55,000
h_i (in IDR)	3,900	4,200	3,300
B_i (in IDR)	26,000	28,000	22,000
m (in years)	2		
z_i	60%		
y (in m^3)	7		

By putting all the data inside equation (27), then the optimum replenishment time (T^*) is 1.5887. After obtaining the value of T^*, then order quantities for Q_A, Q_B, and Q_C are 79, 88, and 71 stripes, respectively. Because the purchase unit from the supplier is in a box unit, then the order quantity in stripes must be divided by the lot size of stripes (s_i) in a box of each type of medicine. Therefore, the order quantities in the box unit for medicines A, B, and C are 8, 9, and 8 boxes, respectively. The order quantity in stripes unit for medicines A, B, and C will become 80, 90, and 80, respectively.

Because the value of shelf life ($m = 2$) is larger than replenishment time ($T^* = 1.5887$), then there will be no expired medicines on the storage. But, if the value of shelf life changes into 1.5 years, then the condition will change, and become a value of shelf life ($m = 1.5$) is smaller than replenishment time ($T^* = 1.5887$). Then, there will be expired quantities, and expired costs will occur.

The optimum space requirements can be calculated by using equation (30) is 6.25 m^3 . Even though the space availability is 7 m^3It is unnecessary to fill the medicines until full because it will make higher total inventory costs and probably expired quantities. So, the suggestion is the remaining space (0.75 m^3) can be used to allocate another medicine.

4.2. Sensitivity Analysis

This section will provide sensitivity based on the results from the calculation above.

The shelf life of medicines determines whether there are any occurrences of expiry medicines or not. If the condition is $m > T^*$, then there will be no expiry medicines and vice versa. If the condition is $m < T^*$, then there will be many occurrences of expiry medicines. The effect of expiry medicines will lead to the occurrences of expiry costs. Figure 4 shows the relationship between shelf life of medicines and total inventory costs prior to space requirements 6.25 m^3 From Figure 4, it shows that total inventory will be higher when the condition is $m < T^*$ and vice versa. Total inventory costs will be lower when satisfying the condition $m > T^*$.

Then, for total inventory costs prior to space requirements, the total inventory cost will decrease until the to the optimum space requirements ($w = 6.25$ m^3) which optimum space requirements result from the optimum order quantity for the medicines. The results can be seen in Figure 5.

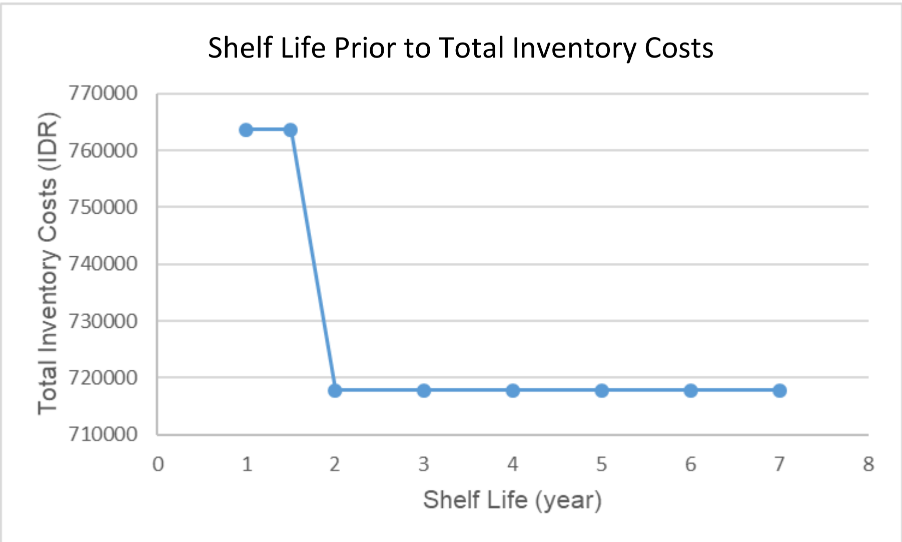

Figure 4. Relationship between Total Inventory Costs and Shelf-Life

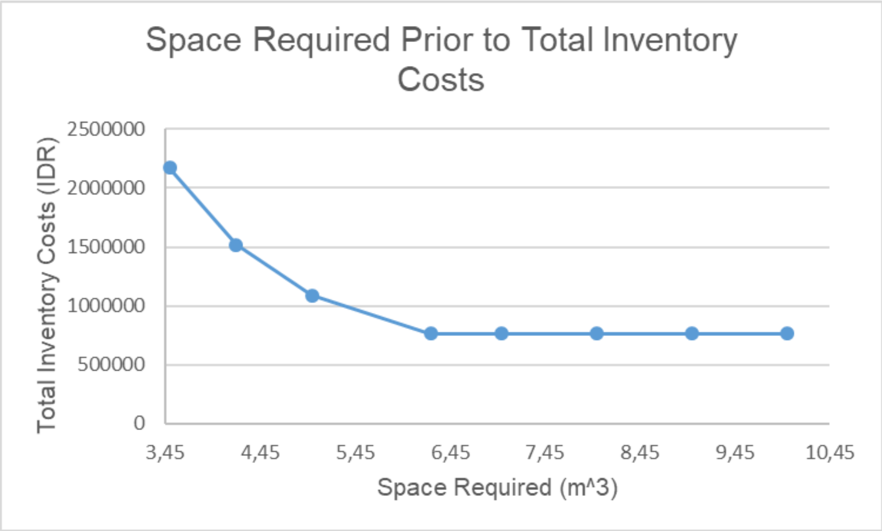

Figure 5. Relationship between Total Inventory Costs and Space Capacity

5. CONCLUSION

Medicines are critical items that must be well controlled by hospitals. If medicines are not being well controlled, then it will cause several disadvantages to hospitals. One of the disadvantages is there will be lots of expenditures that must be spent by hospitals. Most of the expenditures that must be spent are holding costs, ordering costs, shortage costs, and expired costs of the medicines. To minimize the expenditures spent by hospitals, the term of inventory control is needed. Based on the analysis by implementing the proposed inventory model, if the proposed T^* and optimum order quantity are applied, it will result in optimum space requirement and the lowest inventory costs. Therefore, hospitals must order medicines based on the optimum order quantity.

Based on the sensitivity analysis, both the shelf life of medicine and space have the same pattern. Both of them have a bound, in which if their value is smaller than the bound, the total inventory cost is significantly increased compared to the optimal one. It is noted that the bound for the shelf life is T^*, and the bound for the space requirements is w.

For further research, the model development should consider several demand distributions, such as normal distribution, exponential distribution, etc., so that in real-life conditions, hospitals can apply the inventory model based on the demand distribution that occurs in each hospital.

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