e-ISSN: 2685-4090 http://ojs.uajy.ac.id/index.php/IJIEEM

Simulation-based Reliability Evaluation of Maintenance the Efficiency of A Repairable System

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ABSTRACT

The aim of this paper is to study the asymptotic behavior of the Arithmetic Reduction of Intensity (ARI) and Arithmetic Reduction of Age (ARA) models as two imperfect maintenance models. These models have been proposed by Doyen & Gaudoin (2011), the failure process with bathtub failure intensity. The maintenance effect is characterized by the change induced by the failure intensity before and after a failure during the degradation period. To simplify the study, the asymptotic properties of the failure process are derived. Then, the asymptotic normality of several maintenance efficiency estimators can be proved in the case where the failure process without maintenance is known. Practically, the coverage rate of the asymptotic confidence intervals issued from those estimators is studied.

DOI: https://doi.org/10.24002/ijieem.v4i2.6233

Keywords: Repairable systems reliability, bathtub failure intensity, imperfect maintenance, estimation, likelihood.

Research Type: Research Paper

Reference to this paper should be made as follows: Krit, M. (2022). Simulation-based reliability evaluation of maintenance the efficiency of a repairable system. ***International Journal of Industrial Engineering and Engineering Management, 4*****(2), 87-92.**

1. INTRODUCTION

Throughout their operational life, the industrial systems are subjected to preventive and corrective maintenance actions. The essential assumptions on maintenance efficiency are known as minimal maintenance and perfect maintenance. Further realistic ideas of maintenance are somewhat intermediary between these two extremes. Various models have already been proposed for imperfect maintenance effects, for example, de Toledo et al. (2015) and Jin et al. (2022). Single a minority imperfect maintenance model have been statistically studied, particularly regarding the estimation of maintenance efficiency. For virtual age models, some empirical studies on maximum likelihood estimators have been published Wang et al. (2021). The entire of these articles is based on simulation results. Recent articles by Xiaoyang et al. (2020) and Teixeira et al. (2021) deal with simulation and theoretical statistical results in a general class of repair models that include the Nunes & Barbosa (2020) models. However, in these articles, authors consider multiple independent and identical systems over a finite time interval.

Thus, the system behavior without maintenance is known, and the failure intensity is then supposed to be a function of the single efficiency parameter. The failure process without maintenance is characterized by the intensity in bathtub form. This form is presented as a superposition of two Non-Homogeneous Poisson Processes (NHPP) and Homogeneous Poisson one (HPP). The maintenance effect is expressed by the change induced by the failure intensity before and after maintenance. The modeling frameworks are proposed and studied in order to generalize several models to the planned preventive maintenance. The generalization of the models suggested makes it possible to integrate the dependence between corrective and preventive maintenance.

For this fact, we try to proceed in the same way as Syamsundar et al. (2021) by introducing in the first place the properties of the maximum likelihood estimator and second place by interest in exposing an explicit estimator. Several works were carried out on parametric statistical inference in imperfect repair models. We refer, for example, to the Si and al. (2018) study in which authors developed a preventive maintenance policy, as the work of Han and al. (2021). For the case of the Arithmetic Reduction of Intensity (ARI) and the Arithmetic

Reduction of Age (ARA) models, we evoke the Chahrour, and al. (2021) works. The numerical results for our study were at the estimate base on the maximization likelihood method and its properties.

The paper is organized as follows: Section 2 discusses the properties of the failure process. Section 3 analyses the failure intensity of first-order asymptotic extension. Section 4 derives the cumulative failure intensity second-order asymptotic extension. Section 5 introduces the maintenance efficiency estimation. A simulation phase is done in section 6. Section 7 presents some discussion. Finally, section 8 gives conclusions.

2. PROPERTIES OF THE FAILURE PROCESS

The basic outline of the ARI model is to ensure that the maintenance effect shoulders on the failure intensity itself. The fundamental idea is at the origin of the Chan and Shaw (1993) work, for whom the maintenance effect is to reduce the failure intensity with quantity proportional to its value right before maintenance. The following failure intensity characterizes this model:

$$\lambda_t = \lambda(t) - \rho \sum_{j=0}^{\mathcal{N}_t} (1-\rho)^j \ \lambda\left(\mathcal{T}_{\mathcal{N}_t - j}\right) \qquad (1)$$

Mendoza and al. (2022) extended the of Chan-Shaw model. The construction principle of their model appeared in two stages. The first stage is to formulate a model similar to Chan-Shaw, for which the maintenance effect is not to reduce the failure intensity but its increase since the last maintenance. In the second stage, the authors have defined the ARI models with memory $m(ARI_m)$ using intensity written in the following way:

$$\lambda_t = \lambda(t) - \rho \sum_{j=0}^{\min\,(m-1, \mathcal{N}_t - 1)} (1-\rho)^j \ \lambda\left(\mathcal{T}_{\mathcal{N}_t - j}\right) \qquad (2)$$

Under these conditions, the model defined by the intensity (1) can thus be called the ARI model with infinite memory (ARI_∞). Figure 1 represents the general appearance of the failure intensity of the ARI_m model. The first fine line represents the initial intensity, and the second, the minimal degradation intensity, $\lambda_{\min}(t)$. This function is defined as the maximum lower limit for the failure intensity. For the ARI_m model, minimal degradation intensity is:

$$\forall t \geq 0, \qquad \lambda_{\min}(t) = (1-\rho)^m \lambda(t) \qquad (3)$$

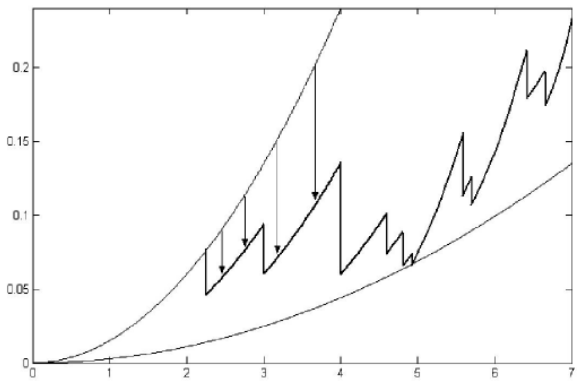

Figure.1 Failure intensity for the ARI_m model

Practically, that means that the examined system is degraded faster than a system whose failure intensity is $\lambda_{\min}(t)$, and slower than a system whose failure intensity is the initial intensity $\lambda(t)$. Near the ARI_m models, the failure intensity is vertically parallel to the initial intensity, as the arrows indicate in Figure 1. The maintenance efficiency is allotted by the estimated value of the parameter ρ, called improvement factor.

The principle of the ARA models is considered in the manner that the maintenance causes to renovate the system. This is with the intuition that its failure intensity at the instant t is equal to the initial intensity at one instant considered as the virtual age of the system, in perception where we consider that the real age of the system put under operation at instant 0 is t. The failure intensity of the ARA model is defined by the following:

$$\lambda_t = \lambda\left(t - \rho \sum_{j=0}^{\min(m-1, \mathcal{N}_t)} (1-\ \rho)^j \mathcal{T}_{\mathcal{N}_t - j} \right) \qquad (4)$$

Figure 2 represents the failure intensity behavior of the ARA_m model. We see that the intensity is parallel to the initial intensity at any instant. Nevertheless, this time the correspondence is horizontal (direction of the arrows). The minimal degradation intensity for the ARA_m model is:

$$\forall t \geq 0, \quad \lambda_{\min}(t) = \lambda((1-\rho)^m t) \qquad (5)$$

The failure intensity, as well for the ARA_m models that for the ARI_m models, expressed using the m last failure instants. Thus, the memory m imitate a Markovian property, remaining the maximum number of failures which can influence the failure intensity.

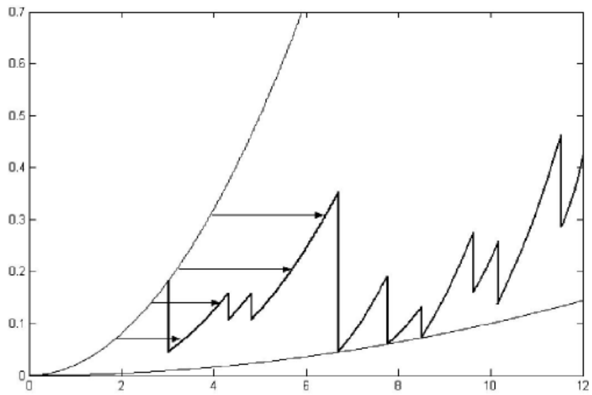

Figure. 2 Failure intensity for the AR, A_m model

3. FAILURE INTENSITY FIRST ORDER ASYMPTOTIC EXTENSION

In this section, the idea is to show that the failure intensity and the asymptotic intensity have identical behavior. Thus, we recall the property presented in Doyen and *al.* (2019), that if exists a function $\lambda_{\min}$, not decreasing and verifies for our model $\forall t > \gamma_1 : \lambda_{\min}(t)$, hence for all $k \geq 0$: $t - \mathcal{T}_{\mathcal{N}_t - k} = o(t)$.

We develop the asymptotic intensity with the same principle followed by Bening (2018), like the minimal degradation intensity for effective maintenance and the maximal degradation intensity for harmful maintenance. Consequently, we admit for our generalization of the ARA_m model, like asymptotic failure intensity, the function: $\lambda_\infty = \lambda((1-\rho)^m(t))$. And the function defined by $\lambda_\infty = (1-\rho)^m\lambda(t)$ is considered asymptotic failure intensity of the ARI_m generalized model.

In continuation, the initial intensity, $\lambda(t)$, is supposed to be a deterministic function, which is not identically null and decreases during the period of the system degradation, defined by Liu et al. (2020) without a maintenance process. These conditions necessarily imply : $\lim_{t\to+\infty} \Lambda(t) = +\infty$. The function Λ is the cumulative failure intensity.If, moreover, λ is a regular variation function, then for $t > \gamma_1$: $\lambda(t) - \lambda(t + o(1)) = o(\lambda(t))$.

Thereafter, the whole asymptotic results of this study rested on a rewriting of the failure intensity, considered exclusively by finished memory models. By means of $\rho\sum_{k=0}^{m-1}(1-\rho)^k = 1-(1-\rho)^m$, this new form of failure intensity is defined, for the ARI_m generalized model, $\forall t \geq \mathcal{T}_m \geq \mathcal{T}_{\gamma_1}$, as:

$$\lambda_t = \lambda_\infty(t) - \frac{\rho}{(1-\rho)^m} \sum_{k=0}^{m-1}(1-\rho)^k\left[\lambda_\infty(t) - \lambda_\infty\left(t + \left(t - \mathcal{T}_{\mathcal{N}_t-k}\right)\right)\right] \quad (6)$$

By means of the foregoing property, this formula is written as follows:

$$\lambda_t = \lambda_\infty(t) - \frac{\rho}{(1-\rho)^m} \sum_{k=0}^{m-1}(1-\rho)^k[\lambda_\infty(t) - \lambda_\infty(t+o(1))] = \lambda_\infty(t) + o(\lambda_\infty(t))$$

In the same way, for the ARA_m generalized models:

$$\lambda_t = \lambda_\infty(t) - \left[\lambda_\infty(t) - \lambda_\infty\left(t + \frac{\rho}{(1-\rho)^m}\sum_{k=0}^{m-1}(1-\rho)^k\left(t - \mathcal{T}_{\mathcal{N}_t-k}\right)\right)\right] \quad (7)$$

And that :

$$\lambda_t = \lambda_\infty(t) - [\lambda_\infty(t) - \lambda_\infty(t+o(1))] = \lambda_\infty(t) + o(\lambda_\infty(t))$$

Consequently, for our generalizations of the ARI_m and ARA_m models, the failure intensity, for all $t > \gamma_1$, verify: $\lambda_t = \lambda_\infty(t) + o(\lambda_\infty(t))$. Under the same conditions, the cumulative failure intensity proves : $\Lambda_t = \Lambda_\infty(t) + o(\Lambda_\infty(t))$. This first order of asymptotic expansion of the failure intensity, make it possible to verify that the increasing phase of the failure intensity and the asymptotic intensity of the ARI_m and ARA_m generalized models of finished memory have same asymptotic behavior.

4. CUMULATIVE FAILURE INTENSITY SECOND-ORDER ASYMPTOTIC EXTENSION

Using the second order of asymptotic expansion of the cumulative failure intensity, Dijoux et al. (2016) go more and express the difference between failure and asymptotic intensities. The author proved the cumulative failure intensity for the ARI_m and ARA_m models with the power failure intensity. In consequence, for our generalization with bath-tub failure intensity, the cumulative failure intensity of the ARI_m model can be written, for all $t \geq \mathcal{T}_m$, as:

$$\Lambda_t = \Lambda_\infty(t) + \frac{\rho}{(1-\rho)^m}\sum_{k=0}^{m-1}(1-\rho)^k \int_{\gamma_1}^{t} \lambda_\infty(s) - \lambda_\infty\left(\mathcal{T}_{\mathcal{N}_s-k}\right)dsz \quad (8)$$

Thereafter, let's suppose that the initial intensity is divergent through the asymptotic intensity, or in an equivalent way, during the degradation phase of the system. That's to say, then, the proposal that the cumulative failure intensity of the ARI_m generalized models ensures:

$$\Lambda_t = \Lambda_\infty(t) + \frac{1-(1+m\rho)(1-\rho)^m}{\rho(1-\rho)^m}\ln\lambda(t) + o\left(\ln\lambda(t)\right) \quad (9)$$

By analogy with the ARI_m generalized models are defined by:

$$\lambda_{]\gamma_1,+\infty}(t) = \frac{1}{\eta_0} + \frac{\beta_2}{\eta_2}\left(\frac{t-\gamma_1}{\eta_2}\right)^{\beta_2-1}$$

Then the cumulative failure intensity of the ARA_m generalized models verify:

$$\Lambda_t = \Lambda_\infty(t) + (\beta_2 - 1)\frac{1-(1+m\rho)(1-\rho)^m}{\rho(1-\rho)^m}\ln(t) + o\left(\ln(t)\right) \quad (10)$$

The two relations (9) and (10) indicate the asymptotic behavior of the failure process of the ARI_m and ARA_m models. In fact, this behavior with finished memory is the same as that of NHPP with intensity λ_∞. Obviously, for the models with finished memory, if $\lambda(t)$ is concave (respectively convex), for the same parameter ρ such as $0 \leq \rho \leq 1$, the asymptotic degradation speed of the ARA_m model is larger (respectively smaller) than that of the ARI_m model. Consequently, ARI_m and ARA_m models with asymptotic intensities having different asymptotic behaviors are similar to NHPP with different failure intensities. This way, ARI_m and ARA_m models with the same parameters are not comparable because they have very different degradation speeds. Nevertheless, it can happen there that if the initial intensity is a functioning power, the values of the maintenance efficiency parameters, such as the models, are comparable. And each of the two models has its own maintenance efficiency.

5. MAINTENANCE EFFICIENCY ESTIMATION

The object now is to study some estimators of maintenance efficiency since the initial intensity is known. In that case, the failure intensity is supposed to depend on a simple parameter ρ. The true value of this parameter will be noted ρ_0. The maximum likelihood estimators MLE of maintenance efficiency, denoted $\hat{\rho}_t^{ML}$. For the ARI_m generalized model maintenance cap$maintenance$intenance efficiency parameter checks:

$$\sqrt{\frac{\Lambda(t)}{(1-\rho_0)^m}}(1-\rho_0)^m - (1-\hat{\rho}_t^{ML})^m \xrightarrow{\mathcal{L}} \mathcal{N}(0,1) \quad (11)$$

efficiency parameter of the ARA_m generalized model, for only one observation of the failure process proves:

$$\sqrt{\frac{(t-\gamma_1)^{\beta_2}}{\eta_2(1-\rho_0)^{m(\beta_2-1)}}}(1-\rho_0)^{m(\beta_2-1)} - (1-\hat{\rho}_t^{ML})^{m(\beta_2-1)} \xrightarrow{\mathcal{L}} \mathcal{N}(0,1) \quad (12)$$

According to the preceding assumptions, we do not know how to prove , that the MLE is convergent when the maximization of likelihood is made on $]-\infty,1]$. So the MLE must be required in compact of $]-\infty,1]$ containing the true value ρ_0 of maintenance efficiency. The explicit estimators (EE), which are not present in this problem type, can exist. These EE verify the same asymptotic properties as the MLE. For the ARI_m generalized model for only one observation of the failure process over $]\gamma_1,t]$, the EE of maintenance efficiency parameter is given by:

$$\hat{\rho}_t^E = 1 - \left[\frac{\mathcal{N}_t}{\Lambda(t)}\right]^{\frac{1}{m}} \quad (13)$$

Similarly, with the ARI_m generalized model, an EE can be defined for the ARA_m generalized model using the initial intensity in bath-tub form. Thus, we define near the last generalization, for only one observation of the failure process, the EE of maintenance efficiency parameter. This estimator is expressed by:

$$\hat{\rho}_t^E = 1 - \left[\frac{\eta_2\mathcal{N}_t}{t^{\beta_2}}\right]^{\frac{1}{[m(\beta_2-1)]}} \quad (14)$$

Considering the asymptotic normality of the estimators introduced in front, we maintain to define the Asymptotic Confidence I, intervals (ACI). It is clear that for the same model, MLE and EE verify the same properties,,They describe the identical ACI. Thus, we can assimilate to the model ARI_m generalized model with finished memory, simultaneously for two estimators the ACI for $(1-\rho_0)^m$ at level δ, given by:

$$ACI(\rho) = (1-\hat{\rho})^m + \frac{u_\delta^2 \pm \sqrt{u_\delta^2[4\Lambda(t)(1-\hat{\rho})^m + u_\delta^2]}}{2\Lambda(t)} \quad (15)$$

where indicates $1-\frac{\delta}{2}$ quantiles of the reduced-centered normal law, $\hat{\rho}$ indicate the MLE or EE and $\Lambda_{]\gamma_1,+\infty}(t) = \frac{1}{\eta_0}t + \left(\frac{t-\gamma_1}{\eta_2}\right)^{\beta_2}$.

Similarly, we can define an ACI for the ARA_m generalized model. The ACI for $(1-\rho_0)^m$ of level δ is defined as follows:

$$ACI(\rho) = (1-\hat{\rho})^{m(\beta_2-1)} + \frac{\eta_2\left(u_\delta^2 \pm \sqrt{u_\delta^2\left[\frac{4}{\eta_2}(t-\lambda_1)^{\beta_2}(1-\hat{\rho})^{m(\beta_2-1)} + u_\delta^2\right]}\right)}{2(t-\lambda_1)^{\beta_2}} \quad (16)$$

6. SIMULATION PHASE

Using simulations groping of the ARI_m and ARA_m models one next to one and for a given ACI, we estimate the coverage Rate (CR). This rate is expressed as the simulations proportion for which the true value of the parameter is in the confidence interval. Obviously, the CR converges to $1-\delta$ when the number of observed failures n increases, where δ represents the ACI threshold. Practically, the CR is a function only of the estimator quality used to build the ACI. Thus, we have estimated over 10000 simulations, the CR of the ACI at level 95 for $m = 1,2$ or 3 , $\beta_1 = 0.75$, $\beta_2 = 3$, $\rho = -1,-0.7,-0.5,-0.2,0,0.2,0.5,0.7$ or 0.9 and $n = 60$. The following notations are used in Figure 3.

$---MLE$	$o\ ARA\_1$	$*\ ARI\_1$
$\cdots\ EE$	$\blacksquare ARA\_2$	$+\ ARI\_2$
	$\times\ ARA\_3$	∇ARI_3

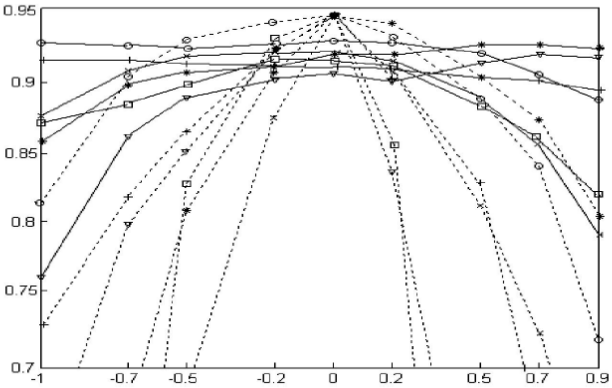

Figure. 3 CR(ρ); β_1 = 0.75; $\beta_2 = 3$; n = 60

Figure 3 represents the CR evolution according to the value of the maintenance efficiency parameter ρ. So the CR of EE depends closely oon the ρ value. The EE provides the most correct ACI for maintenance efficiency close to the minimal case (ρ near to 0), even for a low value of the number of observed failures. On the other hand, when maintenance efficiency is too different from the minimal assumption, the CR converges less quickly, especially in the EE case. This result is a consequence due to the EE, which is founded on an equivalence property between cumulative failure and asymptotic intensities. This equivalence relation is made with a near remainder which is asymptotically equal to:

$$r(t) = \frac{(\beta_2 - 1)}{(1-\rho)^m} \frac{1-(1+m\rho)(1-\rho)^m}{\rho} \ln(t-\gamma_1) \quad (17)$$

At a certain in instan1 and for $\rho = 0$, this quantity is null by hypothesis. In fact, in this case $\Lambda_t = \Lambda_\infty(t)$ whereas, when ρ tends to 1 by lower values, the above difference diverges. Finally, since the maintenance efficiency is degraded and becomes more and more harmful, the cumulative asymptotic intensity increase, and the difference $r(t)$ tends to a constant limit equal to $m(\beta_2 - 1)$.

The MLE are characterized by CR, which are less sensitive to the value of ρ, but it is always under the assumption of minimal maintenance efficiency that the estimators are most correct. This CR behavior whether through of the MLE or EE, can be owed to the operation of the system in the improvement and service life periods, and is maintained by minimal maintenance actions. It appears clearly, on the one hand for low numbers of failures, and on the other hand for the models with high enough memories. Thus, for a great number of failures the ACI are the good approximations for the practical value of maintenance efficiency.

7. DISCUSSION

This paper analyzes the imperfect repair model with the Buth-tub Weibull failure distribution under the asyptotic assumption. We develop the marginal distributions of effective ages and inter-failure times and show a stationary state's existence. When the model reaches this state, the effective ages and the inter-failure times converge towards its limit distributions. The results are then applied to propose a static, dynamic and failure-limit maintenance policy. Numerical simulations are presented to illustrate the policies.

In all applications, preventive maintenance and corrective maintenance are assumed to be equally effective, but this is rarely the case in practice. The use of the virtual age is extremely practical because the models take the efficiency of maintenance into account and make it very easy to carry out Monte Carlo simulations and numerical calculations. The results are consistent with those found in the work Joshi et al. (2019); and Ruiz-Hernández et al. (2020).

Indeed, the failure limit policy is the most advantageous policy but it is also the most difficult to extend because it is necessary to measure the virtual age of the system, not the real age. The dynamic policy is not as efficient as the infinite horizon static policy but it is locally optimal. Moreover, in practice, the computation time to obtain the optimal solution is on the order of milliseconds, which is an order of magnitude different from the time until the planned preventive maintenance.

We can emphasize that estimating the parameters is an important and crucial step in analyzing a set of maintenance data. Inference in the presence of virtual age assumptions has been well discussed in the literature (Doyen & Gaudoin, 2004; Liu et al., 2020) by considering one or more types of interviews. The quality of ML estimators for arithmetic age reduction models has been discussed explicitly in Nguyen et al. (2016). As our article does not focus on inference procedures, we have not provided detailed details on how to estimate the model parameters.

8. CONCLUSION

In this paper, we generalized two classes of imperfect maintenance models using failure intensity in bath-tub shape. We gave new results on our generalizations of arithmetic reduction of age or intensity with memory m. We have shown that *ARIm* and *ARAm* models with finite memories are adapted to reparable systems. In fact, they are asymptotically equivalent to a non-homogeneous Poisson process with no decreasing failure intensity. Their failure process is characterized by equivalence between cumulative failure intensity and cumulative asymptotic intensity.

In the application, it is proposed that if the initial intensity is unidentified, then an estimate could be used. Except this also guides to a different property of the estimator of maintenance efficiency. Further study can be done to extend the statistical properties to this case. For the ARI and ARA generalized models with finished memory, we proposed the explicit estimators of the maintenance efficiency parameter. Then, we presented theoretical statistical results for the estimate of maintenance efficiency. The convergence properties relative to maximum likelihood and explicit estimators were derived. Thus, we could assume that the asymptotic confidence intervals are issued from those estimators.

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