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# Low Wear Rate Selection of Nylon 6-Boron Nitride (PA6/BN) Composite During Composite Development Using Grey Relational Analysis Through the Direct and Indirect Factors of Taguchi Method

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**ABSTRACT**

Wear performance has been evaluated for mechanical equipment using normal load, sliding speed, and sliding distance, but aspect ratios have been traditionally ignored in the literature. Also, limited studies have analyzed wear performance with sparse information. In this study, a grey relational analysis (GRA) technique is proposed for the wear performance analysis of nylon 6/boron nitride composite using aspect ratios. A complete divergence is made from the literature where the aspect ratios of the particulate weight of the composite, normal load, sliding speed, and sliding distance are treated in direct and aspect ratios of 12 cases where the reciprocals of factors, their squares and cubes are considered. Results show that the proposed method of GRA is feasible and offers an adequate illustration of the indices of the parameters of the wear process as opposed to the present method of Taguchi that exists in the literature. A key result is from case 2, which shows that experimental trial 9 with the grey relational grade of 1.00 has the lowest wear rate. The corresponding values of the parameters are 0.05 of the 1/NL parameter, while the SD parameter is 500. This is interpreted as 0.05N-1 of the reciprocal of normal load and 500m for the sliding distance. The principal contribution of this research is the introduction of the grey relational analysis to reduce the wear rate of nylon 6-boron nitride composite. The proposed method is useful as a planning tool for the maintenance engineer to monitor the health of equipment in practice.

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## 1. INTRODUCTION

Globally, manufacturing concerns with rotary machines often experience the wear of machines, leading to ineffective service delivery due to machine breakdown (Pujar et al., 2021). The concerned components, which could be gears and shafts, among others, interact with their environments (i.e., air, liquid, and other parts), and their interacting surfaces are often in relative motion, thus triggering friction and the effort to overcome it using different strategies (Pujar et al., 2021).

The idea is that a wear-resistant material for use in the manufacturing firm ought to process minimum weight, hardness, and maximum flexural rigidity (Hribersek &

Kulovec, 2022). This makes the nylon 6/boron nitride composite a possible candidate for choice and testing (Huang *et al.*, 2020). With this material, it is often detected that infrequently, wear contacts of structural members exist. It may also be a poorly planned preventive maintenance and lubrication systems that exist in the manufacturing plant. Thus, there is a great difficulty for the operators of such manufacturing plants to be efficient and effective. The results are unplanned corrective maintenance activities, escalated maintenance costs and high unit production costs of the outputs of the manufacturing industry. If this problem is not tackled at present, it could lead to complicated unmanageable problems in the future operations of the manufacturing plant. In Kumar & Reddy (2020), four parameters influencing the wear losses of equipment were suggested as the weight of particulate of the additive element of Boron Nitride in the composite, sliding speed, normal load and sliding distance. Unfortunately, the order of importance of these parameters one to another was not stated in the article of reference. This may be because the wear literature currently lacks helpful methods that could aid in distinguishing the importance of one parameter from the other. However, a parallel study of the literature reveals that ignorance of the positioning and relative importance of one parameter against the other has severe negative impacts. One of these impacts is the conflicts that may occur between operators of equipment about the number of resources that should be allocated to them. While intuition is being used presently, a wrong distribution of resources and favoritism may prevail. Here, a needy operator may be denied resources while another operator who is not in need may have the resources in excess and wastefully use them.

Furthermore, in manufacturing industries monitoring and controlling the interactions of major equipment with rotary parts such as gears and shafts within the budget of the industry is a critical challenge as it requires mastery of the equipment wear process, friction generated between moving parts, lubrication effectiveness and design-associated issues. The present monitoring and controlling activities of wear in manufacturing industries have their grounds set on intuition and the use of condition monitoring devices such as oil debris analysis. The presence of significant debris in used oil analysis and further screening may be indicative of an increased wear activity for the components under investigation. This monitoring and controlling approach are widely adopted by managers since their attention is on obtaining optimal results despite the limited available information to these managers.

However, the literature on wear and maintenance activities in the industry has revealed that the above-mentioned monitoring, controlling and intuition practices in wear management may lead to incorrect decisions since the relative importance of the parameters and the optimal experiment may not be used in decision-making. Therefore, optimization of wear parameters and the ordering of parameters in their importance scale has been suggested as a prominent problem to solve in wear control decision-making. However, to the best of the authors' knowledge, previous studies have been deficient in tackling the optimization problem concerning wear losses of nylon 6/boron nitride composite and also short in evolving an order for the parameters of the wear process. While optimizing the parameters of wear is a principal element in improving the product lifespan of nylon 6/boron nitride during operations ordering of parameters according to importance assures that the conflict that may exist between operators in resource sharing is eliminated. Besides, the consideration of the paucity of wear performance of nylon 6/boron nitride, makes it a fertile ground to apply the grey relational analysis to best describe the optimal values of the parameters during experimental counts (Onyegiri & Oke, 2017; Ajibade *et al.*, 2018; Ighravwe *et al.*, 2018; Okponyia & Oke, 2021).

Furthermore, in the literature on performance analysis of worn composites, a recent and similar work to the present one is the wear performance of nylon 6-boron nitride (PA6/BN) composite. In the work, a unique attempt at diverging from the traditional use of parameters to incorporate aspect ratios of parameters was made. Aspect ratios of four parameters, including the reciprocals of weight, normal load, sliding speed and sliding distance were explored to obtain optimization measures for the composite wear performance. Though the philosophy of developing these direct and aspect ratios in Adekoya *et al.* (2023) and the present study is the same, the present work is different from Adekoya *et al.* (2023) in the following respects. Interestingly, direct factors of the Taguchi method are the parameters of the wear process that influence wear directly e.g. normal load and sliding distance. These direct parameters reveal the influence of one parameter on another when not transmitted by a third parameter in terms of ratios. However, indirect parameters influence another parameter indirectly by hanging other factors. The Taguchi method tackles the problem of variances in the measurement of the wear process data. However, it ignores the possibility of tracking the correlation measurement among the wear process parameters. However, the grey relational analysis maps out the system degree claim concerning managing the complex and dynamic characteristics of the parameters and working perfectly in the absence of complete information. This is done by establishing the correlation measures among the parameters. With this information, it offers a structure to decide which analysis warrants investment in research efforts.

In this article, the grey relational analysis is proposed as a method to tackle the complexity and the dynamic attributes of multiple inputs and outputs. This is done by establishing correlation measures of two parameters at a time. It also adds value to the wear evaluation process quantity of the complexity and dynamic nature of the absorption process. The principal contributions of this article are as follows:

1. The method accommodates many inputs and outputs of the wear process and takes on the crisp numeric values in their evaluation of correlation among the parameters. though the direct parameters fundamental to the grey relational analysis are the shading distance and normal load. the grey relational

method accommodates the pairing of direct parameters and indirect parameters. it also contains pairs of indirect parameters. so, shading distance, normal load, their reciprocals, the square of their reciprocals and cubs of their reciprocals are paired in an orderly manner and their correlations are determined.

2. The complex system is analyzed simply with correlation results guiding the engineer on the choice of parameters and their combinations to achieve the best experimental runs upon which the relative performance of the parameters are measured during implication.

## 2. LITERATURE REVIEW

### 2.1. General

In this section, the literature is reviewed as follows: Boopathy *et al.* (2022) analyze the mechanical properties of polymer composite. More specifically, nylon 6 fortified with 5, 15 and 25 weight of silicon carbide and another composition of nylon 6 fortified with 5, 15 and 25 weight boron nitride were produced which the process parameter of injection molding was optimized using the Taguchi method. The parameter considered is the injection pressure while the outputs were the least strength and its distortion temperature. These parameters are at variance with those analyzed in the present studies which are normal load and sliding distance in the direct and indirect forms. Furthermore, Gao *et al.* (2021) analyzed the tribological accomplishment of nylon 6 composite coatings. The parameters include the weight of oil-loaded microcapsules and the frictional temperature of the composite. The outputs are the frictional coefficient and specific wear rate. Compared with the present article, there is a diversion of interest to normal load, sliding distance and the reciprocals. Moreover, Pujar *et al.* (2021) analyzed the essence to investigate the mechanical and wear conduct of the thermoset polymers with their composite and discovered its incessant usage in various aspects of engineering applications. It further reveals the current research and recommendations extracted on mechanical and tribological behaviors and various parameters that control the properties of the polymer-based composite. Besides, an enormous effect has been placed on various ranges of secondary fillers strengthened in polymer to improve mechanical and wear performance. Next, Hribersek *et al.* (2022) use the wear conduct of gear pairs POM-PA66 incorporated with 30% of glass fibers with internal oil in an experiment. It was discovered that wear advancement is associated with elastic material properties linked to higher elongation and coefficient of continuous thermal expansion for POM material in similitude to PA deployed materials. Thus, wear and distortion of gear are inversely proportional to toughness.

Also, Li *et al.* (2017) analyzed the impact of doses of nitrogen ions on titanium and its alloys by plasma immersion ion implantation. Its outcome showed that torsional pine administration and destruction behavior of the titanium and its alloys are heavily related to the dose of the implanted nitrogen ions and the angular displacement amplitude to produce oxidative wear, abrasive wear etc. Furthermore, Singh & Singh (2017) posited that the accomplishment and toughness of the polymer gears lie in their application. It establishes the various processes deployed for the improvement help to reduce the wear of the material of the gears. Thus, the paper emphasizes the impact of polymer composite materials and the idea of blended gears in its development. Moreover, Hutchings & Shipway (2017) noted that corrosive wear can be generated by compact particles sliding on mushy hard surfaces and loosened material. A series of research was carried out to differentiate between sliding particles and the eroded surface of the solid. This research helps to assert that proficiency of distortion of wear can further be enhanced by other forms implanted in a mushy matrix. In addition, Pawar & Abhang (2022) use polymer-on-polymer connections in the design of machines and gadgets. Optimal parametric settings were analyzed through the use of the ANOVA and S/N ratio concerning each factor.

Moreover, Dasari *et al.* (2009) also analyzed the behavior of polymers for their mechanical relevance by the inclusion of a minute percentage of inelastic nanoparticles in the polymers to develop an enhanced mechanical performance in terms of bendness and strength. Furthermore, Myshkin *et al.* (2005) looked into the tribological review of polymers and revealed how velocity load and temperature affected friction. Moreover, Friedrich (2018) focused on the significance of polymer tribology, its distinctive design principles for lower friction and wear concerning sliding against plane metallic correspondent and the synergistic consequence of nanoparticles and conventional fillers and fibers for an absolute tribological performance. Consequent to the above premises, the article reviews the convectional implementation of polymeric tribe-composite in mechanical and other engineering environments. Also Bahadur *et al.* (1995) researched scrubbed and unscrubbed pin surfaces and the movement of film of nylon on 15% CuS and 30% CF composite formed on steel disk surface as being carried out by X-ray photoelectron spectroscopy, secondary ion mass spectroscopy, scanning electron microscopy and energy dispersive X-ray spectroscopy (EDS). This report showed that CuS and nylon deteriorate throughout scrubbing to process Cu and its other forms.

Furthermore, Zhang *et al.* (2021) worked on the unrecognized importance of roughness directionality to polymer wear and posit its research on solid lubricant (5wt% $Al_2O_5$ – PTEE) as a framework polymer to study the outcome of the coarseness invariance on polymer wear. It discovered that (1) all textures produce immense polymer wear relative to the untextured surface. (2) flat groove texture process less wear of the invariance and polymer. Bose & Mahawar (2004) focused on the consequence of connecting mica with varying particulate size on the mechanical, thermal, electrical and microstructural properties of nylon 6. In addition, Mistry & Randhawa (2020) scrutinized the impact of hexagonal boron nitride (h-BN) particles on PA66 composites' mechanical and tribological properties. The output exhibited an enhancement in the modulus of elasticity and

Rockwell hardness of the composites evaluated. Lastly, Khalaj *et al.* (2020) work on the use of hexagonal boron nitride (h-BN) as a composite material with high thermal conductivity and low thermal expansion coefficient. Thus, substantial studies have been conducted to enhance the thermal conductivity of these distinct matrices of the h-BN fillers which are considered in this work. Nonetheless, the evaluation of this material in cases of sparse information and using aspect ratios has been downplayed.

### 2.2. Summary of the literature

In the literature review conducted, several observations were made as follows:

1. Authors conveniently treat mechanical property analysis of the polymer composites alongside their wear property assessment.
2. A wide range of polymer composites has been studied, including nylon-6-6-silicon carbide, composite, nylon-6-graphene oxide/paraffin wax composite, thermoset polymers composite, POM–PA66, nylon-CUS and nylon-CF composite, PA66 and PA66/h- BN composites.
3. In the wear performance analysis domain, the key parameters considered are the normal load, sliding speed and sliding distance. Only recently were aspect ratios introduced. Let these aspect ratios have not been extensively tested and verified to be correct.
4. There is extremely, scarce literature on wear performance that considers the assessment of wear characteristics of composite where the experimental data is sparse. Some models thrive in such a situation and the grey relational analysis has been considered successful in this regard.
5. Most studies on grey relational analysis have omitted a concurrent treatment of the method with aspect ratios of parameters.
6. The prevalent mathematical model of wear is Archards' wear, model.

To extend this model, authors have developed regression models to evaluate the wear performance problem. Notwithstanding, simple linear models have been largely used while non-linear methods are hardly used.

## 3. METHOD

### 3.1 Taguchi method and grey relational analysis

The Taguchi method is one of the two pillar methods used in this work while the grey relational analysis is the second pillar method. Taguchi method was deployed to solve the wear problem for two reasons. First, it is efficient in eliminating and reducing variances and its deployment to the wear optimization problem ensures that this is achieved in the operation before they occur. Moreover, the excellence in operational performance of the wear process is guaranteed. Although Genichi Taguchi, who founded the Taguchi method initially applied it to manufacturing processes, the abundance of evidence for its success in ear-related issues assures its potential success in the present article to lower the wear development process. In this way, the wear process product development will be more efficient since the details obtained from the Taguchi method aid in the quick identification of the most influential parameters in the wear processes. With these details, time could be saved during maintenance (at the lifetime of the machine, for instance) and resources would be conserved.

Furthermore, the grey relational analysis (GRA) is found to be potentially useful in solving the emerging growing concern about the progressive degradation of parts used in the equipment for the manufacturing process and other areas and their potential economic effects on the organization. The GRA was proposed by Denga Julong who was associated with the Huazhong University of Science and Technology. It is one of the elements of the grey system theory as there are numerous theories proposed on the subject to solve diverse problems. In the present situation, the GRA applied in this work is based on a particular idea of information which explains circumstances where information may not be available and those where information is perfectly available as black and white, respectively. This is the situation with the wear process at the fabrication stage of the part where no operational data is known, and the available information may be regarded as black.

### 3.2. Procedure for the adoption of grey relational analysis

The procedure for the adoption of grey relational analysis is well-known and extensively discussed in several articles including Ighravwe *et al.* (2018), Okponyia & Oke (2021), Onyegiri & Oke (2017), and Ajibade *et al.* (2018). In summary, the grey relational analysis uses the grey system theory and the idea of information availability. In this respect when a system such as the wear performance system lacks information, it is described as being black while when the system has full information, it is described as being white. Thus, blackness or whiteness are relative measures of the quality of information that the Grey relational analysis processes. In the wear performance system where grey relational analysis is used, it is assumed that the information obtainable is not perfect and hence, a paucity of information exists. However, with this viewpoint, one is still capable of providing feedback on the system despite the paucity of information available. In the steps that follow, the important steps that will guide the implementation of a grey relational analysis for the wear and performance analysis of nylon 6/boron nitride composite are elaborated.

Step 1: Obtain the parameters for the analysis

Wear parameters are elements of the wear process that are crucial in establishing the performance of nylon 6/boron nitride composite when tested in the wear rig. By identifying the essential parameters of wear in the situation considered, the ease of measurement is attained as feedback on measures provides a means of control through which in updating of the experiment can be done. From this understanding, it is essential to obtain the critical parameter that will represent the wear performance evaluation system when considering nylon 6/boron nitride composite.

Step 2: Establish the number of experimental trials for the process by stating the factors and levels of the parameters

Step 3: Determine the maximum and minimum value of each parameter to be used for the calculation of the normalized sequence:
In establishing the maximum and minimum values, the various values of each parameter are noted and the highest is an indication of the upper boundary for the parameter, while the lowest is an indication of the lower boundary of the parameter.

Step 4: Calculate the normalization sequence with the aid of Step 3 above applying the smaller the better method.

$$NS = \frac{\Delta \max - X_i}{\Delta \max - \Delta \min} \quad (1)$$

Step 5: Compute the deviation sequence by subtracting the value derived on the normalized sequence from the overall maximum value attained on Step 4.

$$DS = \Delta \max - X_i \quad (2)$$

Step 6: Compute the Grey relational coefficient.

$$\text{Grey Coefficient} = \frac{\Delta \min + \xi \Delta \max}{X_i + \xi \Delta \max} \quad (3)$$

where $\Delta \max = 1, \xi = 0.5, \Delta \min = 0$

Notice that two parameters are involved in the evaluation notably the normal load and the sliding distance. These parameters are used to evaluate the grey relational coefficient which is carried out by the addition of the minimum value of the outcome of the experimental trials with the product of the epsilon and the maximum value of the outcome of the experimental trials. The result here is divided by the sum of a changing variable Xi and the product of the epsilon with the differences in the maximum value from the minimum value of a given delta set.

Step 7: Compute the Grey relational grade by finding the averages of the calculated grey relational coefficient established above

$$\gamma_i = \frac{1}{n}\sum_{k=1}^{n} \xi_i(k) \quad (4)$$

At this stage, the computation of the grey relational grade is made. This is achieved by first finding the averages of the grey relational coefficient established in Equation (2) on achieving the process. Equation (4) is developed.

Step 8: State the rank of each parameter deduced from the Grey relational grade above:
In grey relational analysis, ranks are an indication of a place in the hierarchy of the grading system, usually determined by consideration of the differences between numerical values for all the experimental trials considered in a wear performance evaluation. Often, the highest value is 1 when the least value is 0 and other experimental trials are placed between these two extremes. Consequently, the outcome is to define the rank for each of the parameters when linked with the experimental trials generated from previous computations. This outcome is referred to as the grey relational grade.

## 4. RESULTS AND DISCUSSIONS

Information on the control parameters and their levels are shown in Table 1a.

### 4.1. Direct and aspect ratio – Case 1

Concerning Table 1b which considers the direct parameter without the aspect ratios, there are nine experimental trials which correspond to be obtained. The smaller the better method was applied to analyze the normalization sequence, deviation sequence, Grey relational coefficient and Grey relational grade to determine the rank in the response table.

The computations of the two orthogonal arrays of the factors were carried out by determining the overall minimum and maximum value of the direct parameters, namely the normal load and sliding distance. Hence, the normalization sequence is computed by deducting the value of the direct factor from the minimum value of the overall factors and the fraction of the difference between

Table 1a. Control parameters and their respective levels (Kumar and Reddy, 2020)

| Parameters | Labels | Level 1 | Level 2 | Level 3 |
|---|---|---|---|---|
| BN, %wt. | A | 4 | 12 | 20 |
| Normal load, N | B | 10 | 15 | 20 |
| Sliding speed, rpm | C | 100 | 200 | 300 |
| Sliding distance, m | D | 500 | 750 | 1000 |

Table 1b. Direct parameters of normal load and sliding distance for case 1

| | | | Normalizing sequence | | Deviation sequence | | Grey relational coefficient | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Trial No | NL | SD | NL | SD | NL | SD | NL | SD | GRG | Rank |
| 1 | 10 | 500 | 1.00 | 1.00 | 0.00 | 0.00 | 1.00 | 1.00 | 1.00 | 1 |
| 2 | 15 | 750 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 4 |
| 3 | 20 | 1000 | 0.00 | 0.00 | 1.00 | 1.00 | 0.33 | 0.33 | 0.33 | 6 |
| 4 | 10 | 1000 | 1.00 | 0.00 | 0.00 | 1.00 | 1.00 | 0.33 | 0.67 | 3 |
| 5 | 15 | 500 | 0.50 | 1.00 | 0.50 | 0.00 | 0.50 | 1.00 | 0.75 | 2 |
| 6 | 20 | 750 | 0.00 | 0.50 | 1.00 | 0.50 | 0.33 | 0.50 | 0.42 | 5 |
| 7 | 10 | 750 | 1.00 | 0.50 | 0.00 | 0.50 | 1.00 | 0.50 | 0.75 | 2 |
| 8 | 15 | 1000 | 0.50 | 0.00 | 0.50 | 1.00 | 0.50 | 0.33 | 0.42 | 7 |
| 9 | 20 | 500 | 0.00 | 1.00 | 1.00 | 0.00 | 0.33 | 1.00 | 0.67 | 3 |
| Max | 20 | 1000 | | | | | | | | |
| Min | 10 | 500 | | | | | | | | |

Key: Normal load - NL, Sliding Distance – SD. This applies to Tables 2 to 13

Table 2. Direct parameter and aspect ratio for case 2

| | | | Normalizing sequence | | Deviation sequence | | Grey relational coefficient | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Trial No | 1/NL | SD | 1/NL | SD | 1/NL | SD | 1/NL | SD | GRG | Rank |
| 1 | 0.10 | 500 | 0.00 | 1.00 | 1.00 | 0.00 | 0.33 | 1.00 | 0.67 | 4 |
| 2 | 0.07 | 750 | 0.67 | 0.50 | 0.33 | 0.50 | 0.60 | 0.50 | 0.55 | 5 |
| 3 | 0.05 | 1000 | 1.00 | 0.00 | 0.00 | 1.00 | 1.00 | 0.33 | 0.67 | 4 |
| 4 | 0.10 | 1000 | 0.00 | 0.00 | 1.00 | 1.00 | 0.33 | 0.33 | 0.33 | 8 |
| 5 | 0.07 | 500 | 0.67 | 1.00 | 0.33 | 0.00 | 0.60 | 1.00 | 0.80 | 2 |
| 6 | 0.05 | 750 | 1.00 | 0.50 | 0.00 | 0.50 | 1.00 | 0.50 | 0.75 | 3 |
| 7 | 0.10 | 750 | 0.00 | 0.50 | 1.00 | 0.50 | 0.33 | 0.50 | 0.42 | 7 |
| 8 | 0.07 | 1000 | 0.67 | 0.00 | 0.33 | 1.00 | 0.60 | 0.33 | 0.47 | 6 |
| 9 | 0.05 | 500 | 1.00 | 1.00 | 0.00 | 0.00 | 1.00 | 1.00 | 1.00 | 1 |
| Max | 0.10 | 1000 | | | | | | | | |
| Min | 0.05 | 500 | | | | | | | | |

the overall maximum and minimum values. It was observed that 1 and 0 are the results obtained as the maximum and minimum values of the normalization sequence. Furthermore, the deviation sequence was computed by subtracting the result obtained from the respective normalization sequence from 1 being the maximum value. This process was carried out on all the other factors to obtain 1 and 0 on both normal load and sliding distance. The Grey relational coefficient is then calculated which gives positive values all through. This is carried out by finding the difference and fraction of the result obtained from the deviation sequence. Lastly, Grey relational grade is obtained by finding the average of the two factors and it was observed that positive values were obtained all as well through the process. A ranking of the factors was then carried out to determine that experimental trial 1 was the most significant with experimental trial 3 the least significant of the process.

### 4.2. Direct and aspect ratio – Case 2

For case 2, the sliding distance is measured as a direct parameter while the reciprocal of the normal load is computed. 0.1 and 0.05 are the maximum and minimum values obtained on the normal load and 1000 and 500 are obtained respectively for the sliding distance to be used for the computation of the normalization sequence.

As obtained in Table 2, the minimum and maximum values obtained on the normalization sequence after computation were 1 and 0 on both factors. Furthermore, 1 and 0 are the results obtained as the minimum and maximum values established on the computation of the deviation sequence. As observed in case 1 with direct parameters, positive values are derived for the Grey relational coefficient. The same positive values were obtained for the Grey relational grade as attained in case 1. Here, experimental trial 9 is the most significant as experimental trial 4 is the least significant. This is a total contradiction to the result obtained in case 1.

### 4.3. Direct and aspect ratio – Case 3

In case 3, as seen in Table 3, the normal load is measured as a direct parameter with the reciprocal of sliding distance evaluated 20 and 10 are maximum and minimum values obtained from the lot of the normal load with 0.002 and 0.001 obtained respectively for the sliding distance. As expected on the normalization sequence after computation, 1 and 0 are obtained as the maximum and minimum result which was also obtained on the deviation sequence on the application of the formulation upon which the computation was carried out. Positive values

Table 3. Direct parameter and aspect ratio for case 3

| | | | Normalizing sequence | | Deviation sequence | | Grey relational coefficient | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Trial No | NL | 1/SD | NL | 1/SD | NL | 1/SD | NL | 1/SD | GRG | Rank |
| 1 | 10 | 0.002 | 1.00 | 0.00 | 0.00 | 1.00 | 1.00 | 0.33 | 0.67 | 4 |
| 2 | 15 | 0.001 | 0.50 | 0.67 | 0.50 | 0.33 | 0.50 | 0.60 | 0.55 | 5 |
| 3 | 20 | 0.001 | 0.00 | 1.00 | 1.00 | 0.00 | 0.33 | 1.00 | 0.67 | 4 |
| 4 | 10 | 0.001 | 1.00 | 1.00 | 0.00 | 0.00 | 1.00 | 1.00 | 1.00 | 1 |
| 5 | 15 | 0.002 | 0.50 | 0.00 | 0.50 | 1.00 | 0.50 | 0.33 | 0.42 | 7 |
| 6 | 20 | 0.001 | 0.00 | 0.67 | 1.00 | 0.33 | 0.33 | 0.60 | 0.47 | 6 |
| 7 | 10 | 0.001 | 1.00 | 0.67 | 0.00 | 0.33 | 1.00 | 0.60 | 0.80 | 2 |
| 8 | 15 | 0.001 | 0.50 | 1.00 | 0.50 | 0.00 | 0.50 | 1.00 | 0.75 | 3 |
| 9 | 20 | 0.002 | 0.00 | 0.00 | 1.00 | 1.00 | 0.33 | 0.33 | 0.33 | 8 |
| Max | 20 | 0.002 | | | | | | | | |
| Min | 10 | 0.001 | | | | | | | | |

Table 4. Direct parameter and aspect ratio for case 4

| | | | Normalizing sequence | | Deviation sequence | | Grey relational coefficient | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Trial No | 1/NL | 1/SD | 1/NL | 1/SD | 1/NL | 1/SD | 1/NL | 1/SD | GRG | Rank |
| 1 | 0.10 | 0.002 | 0.00 | 0.00 | 1.00 | 1.00 | 0.33 | 0.33 | 0.33 | 7 |
| 2 | 0.07 | 0.001 | 0.67 | -0.08 | 0.33 | 1.08 | 0.60 | 0.32 | 0.46 | 5 |
| 3 | 0.05 | 0.001 | 1.00 | -0.13 | 0.00 | 1.13 | 1.00 | 0.31 | 0.65 | 3 |
| 4 | 0.10 | 0.001 | 0.00 | -0.13 | 1.00 | 1.13 | 0.33 | 0.31 | 0.32 | 8 |
| 5 | 0.07 | 0.002 | 0.67 | 0.00 | 0.33 | 1.00 | 0.60 | 0.33 | 0.47 | 4 |
| 6 | 0.05 | 0.001 | 1.00 | -0.08 | 0.00 | 1.08 | 1.00 | 0.32 | 0.66 | 2 |
| 7 | 0.10 | 0.001 | 0.00 | -0.08 | 1.00 | 1.08 | 0.33 | 0.32 | 0.32 | 8 |
| 8 | 0.07 | 0.001 | 0.67 | -0.13 | 0.33 | 1.13 | 0.60 | 0.31 | 0.45 | 6 |
| 9 | 0.05 | 0.002 | 1.00 | 0.00 | 0.00 | 1.00 | 1.00 | 0.33 | 0.67 | 1 |
| Max | 0.10 | 0.002 | | | | | | | | |
| Min | 0.05 | 0.001 | | | | | | | | |

were recorded for both the two factors for the Grey relational coefficient.

The Grey Relational Grade ranges between 0.3333 and 1 being the maximum and minimum values respectively. Experimental trial 9 is the most important with experimental trial 4, the least appreciated. It was also observed that experimental trials 2 and 8 are in consilient with case 1.

### 4.4. Direct and aspect ratio – Case 4

Here, the reciprocal of the parameters was measured, and nine experimental trials were considered. 0.1 and 0.05 are the maximum and minimum values of the reciprocal of normal load with 0.002 and 0.001 respectively representing the reciprocal of the sliding distance. As seen in Table 4, these values were used to compute the normalization sequence when the current value was deducted from the overall minimum value of the reciprocal of normal load and the division of the difference of the maximum and minimum values were computed to give 0 and 1 respectively for the normalized sequence of reciprocal of normal load and sliding distance. 0.0833 and -0.125 respectively were computed for the deviation sequence when each reciprocal value of the normalized sequence was deducted from 1 being the highest (maximum value).

A similar operation was carried out on the Grey Relational Coefficient when 1, 0 and 0.5 were taken as maximum, minimum and constant values to get 0.3333 and 1 respectively for the reciprocal of the normal load and sliding distance. The Grey Relational Grade (GRG) was then computed using the average of the grey Relational coefficient of normal load and sliding distance to get 0.3333 and this process was applied to all other experimental trials. Lastly, the GRG was onward rank to get the position from 1 – 9 respectively. As expected, experimental trials 2 and 8 equally agreed with case 1.

### 4.5. Direct and aspect ratio – Case 5

A further variation of the direct parameter was carried out in this case. Here, as seen in Table 5, the direct parameter of the sliding distance was maintained with the normal load being squared. This results in the maximum and minimum values of the normal load being 400 and 100 respectively. 1000 and 500 were computed for the sliding distance. The normalized sequence was computed to give between 1 and 0 as its maximum and minimum values. These 1 and 0 can equally be observed as the range of the Minimum and maximum values computed on the deviation sequence.

Additionally, the Grey Relational coefficient (GRC) of the process was carried out by applying the constant

Table 5. Direct parameter and aspect ratio for case 5

| | | | Normalizing sequence | | Deviation sequence | | Grey relational coefficient | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Trial No | $NL^2$ | SD | $NL^2$ | SD | $NL^2$ | SD | $NL^2$ | SD | GRG | Rank |
| 1 | 100 | 500 | 1.00 | 1.00 | 0.00 | 0.00 | 1.00 | 1.00 | 1.00 | 1 |
| 2 | 225 | 750 | 0.58 | 0.50 | 0.42 | 0.50 | 0.55 | 0.5 | 0.52 | 5 |
| 3 | 400 | 1000 | 0.00 | 0.00 | 1.00 | 1.00 | 0.33 | 0.33 | 0.33 | 8 |
| 4 | 100 | 1000 | 1.00 | 0.00 | 0.00 | 1.00 | 1.00 | 0.33 | 0.67 | 4 |
| 5 | 225 | 500 | 0.58 | 1.00 | 0.42 | 0.00 | 0.55 | 1.00 | 0.77 | 2 |
| 6 | 400 | 750 | 0.00 | 0.50 | 1.00 | 0.50 | 0.33 | 0.50 | 0.42 | 7 |
| 7 | 100 | 750 | 1.00 | 0.50 | 0.00 | 0.50 | 1.00 | 0.50 | 0.75 | 3 |
| 8 | 225 | 1000 | 0.58 | 0.00 | 0.42 | 1.00 | 0.55 | 0.33 | 0.44 | 6 |
| 9 | 400 | 500 | 0.00 | 1.00 | 1.00 | 0.00 | 0.33 | 1.00 | 0.67 | 4 |
| Max | 400 | 1000 | | | | | | | | |
| Min | 100 | 500 | | | | | | | | |

Table 6. Direct parameter and aspect ratio for case 6

| | | | Normalizing sequence | | Deviation sequence | | Grey relational coefficient | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Trial No | NL | $SD^2$ | NL | $SD^2$ | NL | $SD^2$ | NL | $SD^2$ | GRG | Rank |
| 1 | 10 | 250000 | 1.00 | 1.00 | 0.00 | 0.00 | 1.00 | 1.00 | 1.00 | 1 |
| 2 | 15 | 562500 | 0.50 | 0.58 | 0.5 | 0.42 | 0.50 | 0.55 | 0.52 | 5 |
| 3 | 20 | 1000000 | 0.00 | 0.00 | 1.00 | 1.00 | 0.33 | 0.33 | 0.33 | 8 |
| 4 | 10 | 1000000 | 1.00 | 0.00 | 0.00 | 1.00 | 1.00 | 0.33 | 0.67 | 4 |
| 5 | 15 | 250000 | 0.50 | 1.00 | 0.50 | 0.00 | 0.50 | 1.00 | 0.75 | 3 |
| 6 | 20 | 562500 | 0.00 | 0.58 | 1.00 | 0.42 | 0.33 | 0.55 | 0.44 | 6 |
| 7 | 10 | 562500 | 1.00 | 0.58 | 0.00 | 0.42 | 1.00 | 0.55 | 0.77 | 2 |
| 8 | 15 | 1000000 | 0.50 | 0.00 | 0.50 | 1.00 | 0.50 | 0.33 | 0.42 | 7 |
| 9 | 20 | 250000 | 0.00 | 1.00 | 1.00 | 0.00 | 0.33 | 1.00 | 0.67 | 3 |
| Max | 20 | 1000000 | | | | | | | | |
| Min | 10 | 250000 | | | | | | | | |

variables to yield values ranging from 0.3333 to 1. The average of the normal load and the sliding distance was then calculated to yield the Grey Relational Grade (GRG) which produces negative values at close range and lies between 0.3333 and 1. It is of interest to state that experimental trial 9 is the most significant followed by experimental trials 5 and 6 with experimental trial 4 being the lowest. Again, experimental trials 2 and 8 bear the same ranking as case 1 with similitude with cases 2, 3 and 4.

### 4.6. Direct and aspect ratio – Case 6

In this case, as seen in Table 6, the direct parameter of normal load was combined with the square of sliding distance. This led to a huge increment in the minimum and maximum value of the sliding distance as 1,000,000 and 250,000 were recorded respectively as their maximum and minimum values. 20 and 10 were retained for the maximum and minimum values for the normal load being in its base state.

On computing the formula for the realization of the normalized sequence, 1 and 0 were recorded for their highest and lowest points. This can equally be observed for the deviation sequence. Positive values were evaluated for the Grey Relational Coefficient which ranges between 0.3333 and 1. Values range of 0.3333 and 1 were equally recorded for the GRG on the computation of its average. The ranking of this case was similar to that of cases 2, 3, 4, and 5 respectively as it retained its positioning of experimental trials 2 and 8 respectively. As stated earlier, there is the correlation of this case with 2, 3, 4 and 5 shows it is in tandem with the previous cases aside from case 1 which had its experimental trial 1 as the most significant.

### 4.7. Direct and aspect ratio – Case 7

In case 7, sliding distance is the direct parameter which is combined with the reciprocal of the square of the normal load. This reciprocal of normal load reduces significantly the maximum and minimum value of the overall experimental trial to 0.01 and 0.0025 respectively. Interestingly, the direct parameter of the sliding distance gives values of 1000 and 500 respectively as the maximum and minimum values for the computation of the Grey Relational process.

Here, as seen in Table 7, the normalized sequence and deviation sequence range between 1 and 0 respectively. A similar trend was noticed on the grey Relational Coefficient with a value range between 0 and 1 for both the normal load and sliding distance. The average of the GRC was computed to evaluate the value of the GRG which ranges between 0.3333 and 1. A trend in support of cases 2, 3, 4, 5, and 6 was noticed in this case. This gives

Table 7. Direct parameter and aspect ratio for case 7

| Trial No | $1/NL^2$ | SD | Normalizing sequence $1/NL^2$ | Normalizing sequence SD | Deviation sequence $1/NL^2$ | Deviation sequence SD | Grey relational coefficient $1/NL^2$ | Grey relational coefficient SD | GRG | Rank |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.010 | 500 | 0.00 | 1.00 | 1.00 | 0.00 | 0.33 | 1.00 | 0.67 | 4 |
| 2 | 0.004 | 750 | 0.74 | 0.50 | 0.26 | 0.50 | 0.66 | 0.50 | 0.58 | 6 |
| 3 | 0.003 | 1000 | 1.00 | 0.00 | 0.00 | 1.00 | 1.00 | 0.33 | 0.67 | 4 |
| 4 | 0.010 | 1000 | 0.00 | 0.00 | 1.00 | 1.00 | 0.33 | 0.33 | 0.33 | 9 |
| 5 | 0.004 | 500 | 0.74 | 1.00 | 0.26 | 0.00 | 0.66 | 1.00 | 0.83 | 2 |
| 6 | 0.003 | 750 | 1.00 | 0.50 | 0.00 | 0.50 | 1.00 | 0.50 | 0.75 | 3 |
| 7 | 0.010 | 750 | 0.00 | 0.50 | 1.00 | 0.50 | 0.33 | 0.50 | 0.42 | 8 |
| 8 | 0.004 | 1000 | 0.74 | 0.00 | 0.26 | 1.00 | 0.66 | 0.33 | 0.50 | 7 |
| 9 | 0.003 | 500 | 1.00 | 1.00 | 0.00 | 0.00 | 1.00 | 1.00 | 1.00 | 1 |
| Max | 0.010 | 1000 | | | | | | | | |
| Min | 0.0025 | 500 | | | | | | | | |

Table 8. Direct parameter and aspect ratio for case 8

| Trial No | NL | $1/SD^2$ | Normalizing sequence NL | Normalizing sequence $1/SD^2$ | Deviation sequence NL | Deviation sequence $1/SD^2$ | Grey relational coefficient NL | Grey relational coefficient $1/SD^2$ | GRG | Rank |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 10 | 0.000004 | 1.00 | 0.00 | 0.00 | 1.00 | 1.00 | 0.33 | 0.67 | 3 |
| 2 | 15 | 1.78E-06 | 0.50 | 0.74 | 0.50 | 0.26 | 0.50 | 0.66 | 0.58 | 5 |
| 3 | 20 | 0.000001 | 0.00 | 1.00 | 1.00 | 0.00 | 0.33 | 1.00 | 0.67 | 3 |
| 4 | 10 | 0.000001 | 1.00 | 1.00 | 0.00 | 0.00 | 1.00 | 1.00 | 1.00 | 1 |
| 5 | 15 | 0.000004 | 0.50 | 0.00 | 0.50 | 1.00 | 0.50 | 0.33 | 0.42 | 7 |
| 6 | 20 | 1.78E-06 | 0.00 | 0.74 | 1.00 | 0.26 | 0.33 | 0.66 | 0.50 | 6 |
| 7 | 10 | 1.78E-06 | 1.00 | 0.74 | 0.00 | 0.26 | 1.00 | 0.66 | 0.83 | 2 |
| 8 | 15 | 0.000001 | 0.50 | 1.00 | 0.50 | 0.00 | 0.50 | 1.00 | 0.75 | 4 |
| 9 | 20 | 0.000004 | 0.00 | 0.00 | 1.00 | 1.00 | 0.33 | 0.33 | 0.33 | 8 |
| Max | 20 | 0.000004 | | | | | | | | |
| Min | 10 | 0.000001 | | | | | | | | |

a clear similitude of the rank obtained on the above cases.

### 4.8. Direct and aspect ratio – Case 8

In this case, a direct parameter of the normal load is measured and combined with the reciprocal of the square of the sliding distance. As seen in Table 8, 20 and 10 were recorded as the maximum and minimum values of the overall experimental trials with 0.000004 and 0.0000001 as the maximum and minimum values of the aspect ratio of the sliding distance.

Similar values were recorded for the normalized sequence and deviation sequence of the normal load and reciprocal of the square of the sliding distance. Expectedly, the GRC of the normal load and the reciprocal of the square of the sliding distance ranges between 0.3333 and 1. On ranking, the equivalent trend was observed in this case. It runs in similitude to cases above aside from case 1 which gives a distinctive difference.

### 4.9. Direct and aspect ratio – Case 9

Here, sliding distance is a direct parameter measured in combination with the cube of normal load. Applying the smaller the better method, the maximum and minimum overall values of the experimental trials are given as 8000 and 1000 for normal load and 1000 and 500 for sliding distance respectively (see Table 9).

This on further processing generated values between 1 and 0 on both instances of the normalized and deviation sequence. The GRC was determined with a value range between 0.3333 and 1 respectively. Ranking of the process was carried out to give a result that corresponds to the cases above namely cases 2, 3, 4, 5, 6, 7 and 8 respectively.

### 4.10. Direct and aspect ratio – Case 10

Case 10 considers a direct parameter concerning the normal load in combination with the use of the sliding distance. 20 and 10 were computed for the maximum and minimum overall value of the experimental trials and 10^9 and 1.25 *10^8 for the maximum and minimum value of the cube of the sliding distance (see Table 10).

On computation, 1 and 0 were observed to be the maximum and minimum derived on application for the derivation of the normalized sequence and deviation sequence respectively. Further computation was carried out to determine the GRC thus giving a value range between 0.3333 and 1. A similar value range as that of the GRC was observed on the computation of the average of the GRC to determine the Grey Relational Grade (GRG). As expected, the ranking on this case was observed to

Table 9. Direct parameter and aspect ratio for case 9

| | | | Normalizing sequence | | Deviation sequence | | Grey relational coefficient | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Trial No | $NL^3$ | SD | $NL^3$ | SD | $NL^3$ | SD | $NL^3$ | SD | GRG | Rank |
| 1 | 1000 | 500 | 1.000 | 1.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 | 1 |
| 2 | 3375 | 750 | 0.661 | 0.500 | 0.339 | 0.500 | 0.596 | 0.500 | 0.548 | 5 |
| 3 | 8000 | 1000 | 0.000 | 0.000 | 1.000 | 1.000 | 0.333 | 0.333 | 0.333 | 8 |
| 4 | 1000 | 1000 | 1.000 | 0.000 | 0.000 | 1.000 | 1.000 | 0.333 | 0.667 | 4 |
| 5 | 3375 | 500 | 0.661 | 1.000 | 0.339 | 0.000 | 0.596 | 1.000 | 0.798 | 2 |
| 6 | 8000 | 750 | 0.000 | 0.500 | 1.000 | 0.500 | 0.333 | 0.500 | 0.417 | 7 |
| 7 | 1000 | 750 | 1.000 | 0.500 | 0.000 | 0.500 | 1.000 | 0.500 | 0.750 | 3 |
| 8 | 3375 | 1000 | 0.661 | 0.000 | 0.339 | 1.000 | 0.596 | 0.333 | 0.464 | 6 |
| 9 | 8000 | 500 | 0.000 | 1.000 | 1.000 | 0.000 | 0.333 | 1.000 | 0.667 | 4 |
| Max | 8000 | 1000 | | | | | | | | |
| Min | 1000 | 500 | | | | | | | | |

Table 10. Direct parameter and aspect ratio for case 10

| | | | Normalizing sequence | | Deviation sequence | | Grey relational coefficient | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Trial No | NL | $SD^3$ | NL | $SD^3$ | NL | $SD^3$ | NL | $SD^3$ | GRG | Rank |
| 1 | 10 | 1.25E+08 | 1.0 | 1.000 | 0.0 | 0.000 | 1.000 | 1.000 | 1.000 | 1 |
| 2 | 15 | 4.22E+08 | 0.5 | 0.661 | 0.5 | 0.339 | 0.500 | 0.596 | 0.548 | 5 |
| 3 | 20 | 1.00E+09 | 0.0 | 0.000 | 1.0 | 1.000 | 0.333 | 0.333 | 0.333 | 8 |
| 4 | 10 | 1.00E+09 | 1.0 | 0.000 | 0.0 | 1.000 | 1.000 | 0.333 | 0.667 | 4 |
| 5 | 15 | 1.25E+08 | 0.5 | 1.000 | 0.5 | 0.000 | 0.500 | 1.000 | 0.750 | 3 |
| 6 | 20 | 4.22E+08 | 0.0 | 0.661 | 1.0 | 0.339 | 0.333 | 0.596 | 0.464 | 6 |
| 7 | 10 | 4.22E+08 | 1.0 | 0.661 | 0.0 | 0.339 | 1.000 | 0.596 | 0.798 | 2 |
| 8 | 15 | 1.00E+09 | 0.5 | 0.000 | 0.5 | 1.000 | 0.500 | 0.333 | 0.417 | 7 |
| 9 | 20 | 1.25E+08 | 0.0 | 1.000 | 1.0 | 0.000 | 0.333 | 1.000 | 0.667 | 4 |
| Max | 20 | 1.00E+09 | | | | | | | | |
| Min | 10 | 1.25E+08 | | | | | | | | |

Table 11. Direct parameter and aspect ratio for case 11

| | | | Normalizing sequence | | Deviation sequence | | Grey relational coefficient | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Trial No | $1/NL^3$ | SD | $1/NL^3$ | SD | $1/NL^3$ | SD | $1/NL^3$ | SD | GRG | Rank |
| 1 | 0.001000 | 500 | 0.000 | 1.0 | 1.000 | 0.000 | 0.333 | 1.000 | 0.667 | 4 |
| 2 | 0.000296 | 750 | 0.804 | 0.5 | 0.196 | 0.500 | 0.719 | 0.500 | 0.609 | 5 |
| 3 | 0.000125 | 1000 | 1.000 | 0.0 | 0.000 | 1.000 | 1.000 | 0.333 | 0.667 | 4 |
| 4 | 0.001000 | 1000 | 0.000 | 0.0 | 1.000 | 1.000 | 0.333 | 0.333 | 0.333 | 8 |
| 5 | 0.000296 | 500 | 0.804 | 1.0 | 0.196 | 0.000 | 0.719 | 1.000 | 0.859 | 2 |
| 6 | 0.000125 | 750 | 1.000 | 0.5 | 0.000 | 0.500 | 1.000 | 0.500 | 0.750 | 3 |
| 7 | 0.001000 | 750 | 0.000 | 0.5 | 1.000 | 0.500 | 0.333 | 0.500 | 0.417 | 7 |
| 8 | 0.000296 | 1000 | 0.804 | 0.0 | 0.196 | 1.000 | 0.719 | 0.333 | 0.526 | 6 |
| 9 | 0.000125 | 500 | 1.000 | 1.0 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 | 1 |
| Max | 0.001000 | 1000 | | | | | | | | |
| Min | 0.000125 | 500 | | | | | | | | |

follow a similar trend to those of cases 2, 3, 4, 5, 6, 7, 8 and 9. This shows the aspect ratios are in agreement with one another.

### 4.11. Direct and aspect ratio – Case 11

In this case, sliding distance is a direct parameter measured and the reciprocal of the cube of normal load. The computation of the overall experimental value gives a maximum and minimum value of 0.001 and 0.0000125 with 1000 and 500 respectively for the reciprocal of the cube of the normal load and sliding distance.

As presented in Table 11, these values were used to compute the normalized sequence. Predictably, 1 and 0 were recorded for the value range of the normalized sequence and on further analysis, the same value range was observed for the deviation sequence. Further computation gives the Grey Relational Coefficient with a value range of 0.3333 and 1. Finding the average of the GRC gives a value range between 0.3333 and 1.

Table 12. Direct parameter and aspect ratio for case 12

| Trial No | NL | $1/SD^3$ | Normalizing sequence NL | Normalizing sequence $1/SD^3$ | Deviation sequence NL | Deviation sequence $1/SD^3$ | Grey relational coefficient NL | Grey relational coefficient $1/SD^3$ | GRG | Rank |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 10 | 8E-09 | 1.0 | 0.000 | 0.0 | 1.000 | 1.000 | 0.333 | 0.667 | 4 |
| 2 | 15 | 2.37E-09 | 0.5 | 0.804 | 0.5 | 0.196 | 0.500 | 0.719 | 0.609 | 5 |
| 3 | 20 | 1E-09 | 0.0 | 1.000 | 1.0 | 0.000 | 0.333 | 1.000 | 0.667 | 4 |
| 4 | 10 | 1E-09 | 1.0 | 1.000 | 0.0 | 0.000 | 1.000 | 1.000 | 1.000 | 1 |
| 5 | 15 | 8E-09 | 0.5 | 0.000 | 0.5 | 1.000 | 0.500 | 0.333 | 0.417 | 7 |
| 6 | 20 | 2.37E-09 | 0.0 | 0.804 | 1.0 | 0.196 | 0.333 | 0.719 | 0.526 | 6 |
| 7 | 10 | 2.37E-09 | 1.0 | 0.804 | 0.0 | 0.196 | 1.000 | 0.719 | 0.859 | 2 |
| 8 | 15 | 1E-09 | 0.5 | 1.000 | 0.5 | 0.000 | 0.500 | 1.000 | 0.750 | 3 |
| 9 | 20 | 8E-09 | 0.0 | 0.000 | 1.0 | 1.000 | 0.333 | 0.333 | 0.333 | 8 |
| Max | 20 | 8E-09 | | | | | | | | |
| Min | 10 | 1E-09 | | | | | | | | |

Table 13. Summary of the best ranks for each case

| Case | 1st parameter | 2nd parameter | Grey relational grade | Comment |
|---|---|---|---|---|
| 1 | NL | SD | 1.00 | Above |
| 2 | 1/NL | SD | 1.00 | Above |
| 3 | NL | 1/SD | 0.33 | Below |
| 4 | 1/NL | 1/SD | 0.67 | Below |
| 5 | $NL^2$ | SD | 0.67 | Below |
| 6 | NL | $SD^2$ | 0.67 | Below |
| 7 | $1/NL^2$ | SD | 1.00 | Above |
| 8 | NL | $1/SD^2$ | 0.33 | Below |
| 9 | $NL^3$ | SD | 0.67 | Below |
| 10 | NL | $SD^3$ | 0.67 | Below |
| 11 | $1/NL^3$ | SD | 1.00 | Above |
| 12 | NL | $SD^2$ | 0.33 | Below |
| | Average | | 0.67 | |

Anticipatedly, case 11 was in tandem with other aspect ratios treated.

### 4.12. Direct and aspect ratio – Case 12

In this situation, the normal load was the direct parameter measured with the reciprocal of the cube of the sliding distance. Of interest is the drastic reduction in the value of its maximum and minimum rate gives 8E-9 and 1E-9 respectively. 20 and 10 are retained for the maximum and minimum overall experimental trial being conserved at its direct level.

Notedly, 1 and 0 were observed on the computation of the normalized sequence and deviation sequence. As presented in Table 12, the GRC was computed to give values between 0.3333 and 1. Expectedly, a pattern similar to those observed in cases 2, 3, 4, 5, 6, 7, 8, 9, 10 and 11 as it experienced the same case.

In this work, the grey relational analysis was conducted for each case to establish which of the experimental trials provides the best rank accompanied by a grey relational grade. It was found that for cases 1 - 12, the following experimental trials produced the best grade: experimental trials 1, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, and 9. Furthermore, it was found that an average of the grey relational analysis could be obtained considering all the twelve cases which yield 0.69 (Table 13). A further step was made to create an extra colon of the grey relational grade. In this situation, comments of "below", "above" or "equal to" to the average values were made. This simply means that the average value was compared with each case. For instance, the grey relational grade for case 1 is 1 but the average relational grade for all twelve cases is 0.69, thus, they attained a grey relational grade for the first rank in case 1 is above, which means that the grey relational grade is greater than the average. By following a similar logic, there are four instances where the considered grey relational grade is above the average and also eight other instances where the considered grey relational grade is below. These are 33.3% and 66.7% for the "above" and "below" respectively. From this result, knowing fully that a higher grey relational grade is desired the lower ones are desired than the lower ones, which means that the majority of the cases have satisfactory grey relational grades.

### 4.13. Low wear rate results

Interestingly, the main objective of the present study is to find the PAL/BN composite with a low wear rate. This was achieved in the various cases as follows: in case 1, experimental trial 1 produced the best performance as it is ranked 1st while experimental trial 3 yielded the worst results of grey relational grade of 1 and 0.33, respectively. Now, the low wear rate is traceable to 10N and 500m (i.e. level 1) of normal load sliding distance, respectively. This information is obtained from Table 1a. This was achieved by using the direct factors alone. Next is case 2, which considers the combination of direct parameters and aspect ratios. In this instance, experimental trial 9 with the grey relational grade of 1.00 is the lowest wear rate. The corresponding values of the parameters are 0.05 of the 1/NL parameter white the SD parameter is 500. This is interpreted as $0.05N^{-1}$ of the reciprocal of normal load and 500m for the sliding distance. Now, in case 3, which contains direct and aspect ratio, the lowest wear rate occurs at experimental trial 4 where the grey relational grade is 1. Here, the parameters are NL and 1/SD as 10 N and $0.001m^{-1}$ for normal load and the reciprocal of the sliding distance, respectively. In case 4, the direct and aspect ratios were treated and the experimental trial 9 with the grey relational grade of 0.67 gives the lowest wear rate. It has 1/NL as $0.05N^{-1}$ with 1/SD as $0.002m^{-1}$, being the reciprocal of the normal load and the reciprocal of the sliding distance, respectively. In case 5, the direct parameter of SD and the aspect ratio of $NL^2$ are considered to give the lowest wear rate of 100m as the sliding distance. This was achieved with experimental trial 1 and a grade relational grade of 1.00. For case 6, the direct parameter NL and the indirect parameter of $SD^2$ yielded the lowest wear rate at experimental trial 1 of 10N for the normal load and 250000 $m^2$ for the square of the sliding distance. In case 7, the direct parameters that were combined with the aspect ratio are the SD and $1/NL^2$, which obtained the highest rank at experimental trial 1, which corresponds to a grey relational grade of 1.00. The associated lowest wear rate has the following parameters 0.003 of $1/NL^2$ and 500 of SD, which is interpreted as $0.003N^{-2}$, being the reciprocal of $NL^2$ and 500m of sliding distance. In case 8, the direct parameter is NL while the indirect parameter is $1/SD^2$, which attained the lowest wear rate at the grey relational grade of 1.00. The associated NL value is a normal load of 10N while the reciprocal of the square of sliding distance, $1/SD^2$, is $0.000001m^{-2}$. For case 9, the direct parameter is SD and the indirect parameter is $NL^3$, which obtained the lowest wear rate at the grey relational grade of 1.00 at the experimental trial 1. The lowest wear rate at this experimental trial 1 is an SD of 500 and an $NL^3$ of 1000, implying a sliding distance of 500m and the cube of the normal load of $1000N^3$. In case 10, the normal load is NL, which is 10N while the $SD^3$, which is $1.25E+08m^3$ are the points of the lowest wear rate, obtainable with experimental trial 1, which has a grey relationship grade of 1.00. For case 11, the lowest wear rate was obtained at experimental trial 9 with a grey relational grade of 1.00. Here, the direct parameter is SD while the indirect parameter is $1/NL^3$, with the lowest wear rate obtained at the parametric threshold of 500m for SD (sliding distance) and $0.000125N^{-3}$ of the reciprocal of $NL^3$, (i.e. normal load). For case 12, the direct parameter is NL while the indirect parameter is $1/SD^3$ and the lowest wear rate was obtained at experimental trial 4, which attained a grey relational grade of 1.00. The corresponding NL value is 10N, which is the normal load while the $1/SD^3$ is 1E-09 $m^{-3}$. The earlier term NL is the direct parameter while the latter term $1/SD^3$ is the indirect parameter

### 4.14. Summary of results

The following are the results arising from the application of the grey relational analysis to the various combinations of direct and indirect factors as well as direct factors alone:

1. Direct factors (case 1): NL versus SD: The parameters NL and SD indicate $NL_i SD_{ii}$ as the best experimental run, which translates to 10N and 500m of normal load and sliding distance, respectively.
2. Direct factors and aspect ratios (case 2): 1/NL versus SD: The results of 1/NL and SD as parameters of the wear process show $1/NL_9$ $SD_9$ as the utmost experimental run, which translates to 0.1N and 500m of normal load and sliding distance, respectively.
3. Direct and aspect ratio (case 3): NL versus 1/SD: By analyzing the results of the combined NL and 1/SD, the best experimental run is 9, which is represented as $NL_9$ $1/SD_9$. It is translated as 10N and 0.002m of normal load and sliding distance, respectively.
4. Direct and aspect ratio (case 4): 1/ NL and 1/ SD: When analyzing the results from the combined 1/NL and 1/SD, it was found that experimental count 9 attains the best threshold at $1/NL_9$ $1/SD_9$. This is translated as 0.1 N and 0.002m of normal load and sliding distance, respectively
5. Direct and aspect ratio (case 5): $NL^2$ versus SD: In analyzing the results of the combined $NL^2$ and SD, the best experimental run was chosen as 9, which is stated as $NL_9^2$ $SD_9$. The translation of this is 100 N and 500 m of normal load and sliding distance, respectively.
6. Direct and aspect ratio (case 6): NL versus $SD^2$: When examining the relationship of NL and $SD^2$ and how it affects the results of the grey relational analysis, it was found that the best experimental is 9, the setting is indicated as $NL_6$ $SD_6$ which is interpreted as 10N and 25000m of normal load and sliding distance, respectively.
7. Direct and aspect ratio (case 7): $1/NL^2$ versus SD: In examining the relationship between $1/NL^2$ and SD and the influence of these factors in producing the optimal results, it was found that the experimental run 9, having the setting of $1/NL_9^2SD_9$ is the best result. This is interpreted as 0.01N and 500m of normal load and sliding distance, respectively.
8. Direct and aspect ratios (case 8): NL versus $1/SD^2$: Here, after calculating the results, the best experimental run emerged as 9. The setting is indicated as $NL_9 1/SD_9^2$, which is interpreted as 10N and 0.000004m of normal load and sliding distance, respectively.

9. Direct and aspect ratio (case 9): $NL^3$versus SD: In this aspect, the best experimental run is 9 when the relationship between $NL^3$ and SD is considered. The setting is written as $NL_9^3$ $SD_9$, which is interpreted as 1000N and 500m of normal load and sliding distance, respectively.
10. Direct and aspect ratio (case 10): NL versus $SD^3$: Parameters NL and $SD^3$ are combined. Experimental trial 9 emerged as the best with the setting of $NL_9SD^3$. This is interpreted as 10N and 125, 000, 000m of normal load and sliding distance, respectively.
11. Direct and aspect ratio (case 11): $1/NL^3$versus SD: Here, $1/NL^3$ is combined best experimental trial is at 9. This means 0.001N and 500m of normal load and sliding distance, respectively. This is interpreted from the experimental setting $(1/NL^3)_9SD_9$.
12. Direct and aspect ratio (case 12): NL versus $1/SD^3$: Here, as NL is combined with $1/SD^3$, the best experimental trial was noted as 9. The setting is given as $NL_9(1/SD^3)_9$, which means 10N and 0.00000008m, interpreted for normal load and sliding distance, respectively.

### 4.15. Comparison of the present study with the literature

Moreover, the previous work of Adekoya *et al.* (2023) is compared to Adekoya *et al.* (2023) which implemented direct and indirect parameters using the Taguchi Pareto method and the same material, PA6/BN composite. In comparing the 18 cases of Adekoya *et al.* (2023) only two cases of 1/NL versus 1/SD and $NL^3$ versus SD appear to be common to the present study. On studying the outcomes of these analyses, it was noted that the results of Adekoya *et al.* (2023) and the answer to the present objective of the study, vis-à-vis the lower wear rate are the same in both cases. This implies that the results presented by Adekoya *et al.* (2023) validate the outcomes obtained in the present study since they are the same.

## 5. CONCLUSION

In this study, the grey relational analysis was performed for removed direct and aspect ratios such as direct factors and the combination of direct factors and squares or cubes of the factors. The ranks of the various experimental counts were ascertained using the various scenarios of combinations. The following conclusions emerge from the study:

(1) Compared with the case of direct factors alone, the following cases perform competitively with the direct factors having a grey relational grade of 1.00. these factors are sliding distance against each of the reciprocal of the normal load, the reciprocal of the square of the normal load and the reciprocal of the cube of the normal load. However, the cases of the reciprocal of sliding distance against normal load or reciprocal of normal load exhibited a grey relational grade below 1.00, which is undesired. The case of square and reciprocal of the square of sliding distance against the normal load shows poor performance in the grey relational grade, which is below 1.00. Thus, tribology engineers should utilize the direct factors of normal load and sliding distance and the sliding distance against reciprocals of the square of normal load and reciprocal of the cube of normal load as standard in tribological measurements.

(2) The use of aspect ratios with the direct factors, particularly for normal load and sliding distance, which was first proposed in Adekoya *et al.* (2023) is feasible and shows reliable results.

(3) Future studies may introduce some other factors such as inflationary and interest rates to create a comprehensive analysis of the factors that influence wear performance when working with PA6/BN composites

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