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Aspect Ratio-based Taguchi Method with An Application to the Friction Stir Welding of AA6062-T6 Alloy

Osita Prince Francis[1]*, Bayo Yemisi Ogunmola[1], Nehemiah Sabinus Alozie[1], Adeyinka Oluwo[1], John Rajan[2], Swaminathan Jose[3], Sunday Ayoola Oke[1], Ayomide Sunday Ibitoye[1]

[1] Department of Mechanical Engineering, University of Lagos, Lagos, Nigeria
[2] Department of Manufacturing Engineering, School of Mechanical Engineering, Vellore Institute of Technology, Vellore, India
[3] School of Mechanical Engineering, Vellore Institute of Technology, Vellore, India

Email: princeofrancis24@gmail.com, bogunmola@gmail.com, ns.alozie@yahoo.com, o.adeyika@yahoo.com, ajohnrajan@gmail.com, swajose@gmail.com, sa_oke@yahoo.com, ayomibitoye400@gmail.com

*Corresponding author

ABSTRACT

This research proposes a new method of modified Taguchi method based on aspect ratios of the parameters integrated with the present worth method for the determination of optimal parametric setting during the friction stir welding process. As a cornerstone feature in the optimization procedure, aspect ratios are uniquely formulated where single parameters are replaced with products of parameters, squares of a particular parameter multiplied by a parameter, and only squares of each parameter information that represent inputs for the determination of the orthogonal matrix, heading to the optimal parametric setting computations, ranks, and delta determination. A wide range of 83 formulations was considered. Unlike previous research, this article accounts for multiple combinations of aspect ratios greater than the members of parameters present in the factor-level framework in the traditional setting of the Taguchi scheme. A principal result reveals that when the parameters were interchanged from A, B, and C to ABC, A^2C, A^2 B, A^2, $B^{2,}$ and C^2, indicating tool till angle, tool rotational speed, and welding speed for A, B and C, respectively, the optimal parametric setting was 462000 (^{0}.rpm.mm/min), 990 (^{0}.mm/min), 12600 (^{0}.rpm.9^0), 1960000rpm, 12100mm/min^2. The result assists welding engineers in implementing optimal decisions during friction stir welding activities. The findings of this study stimulate welding engineers to establish sources of poor-quality welds and optimize the outputs while reducing welding costs.

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1. INTRODUCTION

Presently, the most research attention on friction stir welding has been in the context of the heat generated during the process. Still, small and increasingly significant research is about the optimization of the process parameters. However, the Taguchi method has been described as a universally accepted tool of optimization for friction stir welding concerns. However, the aspect ratios are found as one of the superior methods over the traditional method of direct factors known for some years as the foundation of the Taguchi method of optimization. Furthermore, the focus on aspect ratios has progressed in recent years. Notwithstanding, the development of the optimal parametric settings, delta values for factors, and their ranks is still a challenging task. Thus, in this article, it is argued that the direct factors should be complemented with the aspect ratios to evaluate the signal-to-noise ratios, the cumulative signal-to-noise ratios, optimal parametric settings, delta values, and ranks of the parameters. The attention towards research on aspect ratios in friction stir welding is increasing intensively because of the wide scope of aspect ratios possible in combinations. The reciprocals, of the parameters, their squares and cubes are emerging parts of the aspect ratio family and studies are ongoing to expand the scope of testing of these parameters.

The significance of this article is as follows. Firstly, this study provides a platform for welding operations decision-makers in light and heavy industries and managers within the friction stir welding industry to review the present standards, which probably deviate from the optimal values regarding parametric implementations. Secondly, limited investigations on friction stir welding regarding the AA6062-T6 alloy have been implemented with specific targets on the tool tilt angle, tool rotational speed, and welding speed, and the approach has been the adoption of direct factors within the Taguchi methodical framework. Hence this article, which employs both the direct factors and aspect ratios in the perspective of testing the Taguchi, Taguchi-Pareto, and Taguchi-ABC frameworks is important in bridging the gap in research. Thirdly, the article analyzed the experimental data through the implementation of the Taguchi, Taguchi-Pareto, and Taguchi-ABC methods, which are methods used for the combined optimization and prioritization of parameters of the friction stir welding process. Hence, the article evaluated the parameters in optimization and placed them in order of importance, thereby indicating what parameters to concentrate efforts on during resource distribution to parameters when attempting to implement the friction stir welding process. The fourth significance is that the literature review suggests that the three methods are useful for planning purposes when considering engineering processes. This is meant to enhance the overall performance of the friction stir welding process. Considered as a whole, the present article offers clear evidence to re-echo the optimization process, particularly with the prioritization of parameters, which is an alarming issue in the welding industry regarding aluminum alloy welding for both lightweight and heavy-weight industries, especially from the perspective of performance improvement.

Furthermore, this present research is focused on the evaluation of the optimal parametric values of the friction stir welding process using the combined direct factors and aspect ratios while considering the prioritization of factors using the Taguchi, Taguchi-Pareto, and Taguchi-ABC methods. The principal parameters considered are the tool tilt angle (TTA), tool rotational speed (TRS), and welding speed (WS). These are the declared direct factors. However, aspect ratios are formed from these factors by considering in turn, the reciprocals of each of the factors. At first, given the TTA, TRS, and WS as the direct parameters, the TTA is used to divide itself, TRS, and WS to obtain unity, TRS/TTA, and WS/TTA. Then in turn, as the TTA is divided by TRS, TRS is divided by itself and WS is divided by TRS, a new set of aspect ratios, namely TTA/TRS, unity, WS/TRS is formed, which adds up the total aspect ratios to four. Finally, on the issue of reciprocal, each of the TTA, TRS, and WS is divided by WS to yield TTA/WS, TRS/WS, and unity. This makes the total aspect ratio to be six. Now, another option is to consider the square of the reciprocals where the denominators are sequenced. In this case, six possible options of aspect ratios are generated, these are TRS/TTA^2, WS/TTA^2, TTA/TRS^2, WS/TRS^2, TTA/WS, and TRS/WS^2. It should be noted that the reciprocals of these factors of TRS^2/TTA, WS^2/TTA, TTA^2/TRS, WS^2/TRS, TTA^2/WS, and TRS^2/WS may still be obtainable. Furthermore, cubes and products of these factors are done, making a total of 81 factors resulting in the aspect ratios.

Now, since previous research on aspect ratios on a different subject from the present subject proved that they are feasible and helpful in improving the optimal parameters of the process, these 81 aspect ratios are considered based on experimental data obtained from Khan (2020). Consequently, as an encouragement from the outcome of previous literature that assures the feasibility of aspect ratio application, it was concluded that studies on aspect ratios of parameters in the optimization of friction stir welding parameters are still in demand. Thus, a comprehensive study on the experimental data provided by Khan (2020) could be useful in filling the research gap in the friction stir welding process regarding aspect ratio deployment.

2. LITERATURE REVIEW

In the literature, several aspects of research on friction stir welding have been reported, which fall into two main categories, including experimental and analytical studies. While district studies are available on either category, some studies combine these two aspects. The most prominent reports are reviewed as follows:

2.1. Aspect ratio application in Taguchi optimization

In engineering optimization, aspect ratios have been introduced as an idea to revolutionize operations and have recently been used in the optimization of turning parameters (Adegoke et al., 2022; Adegoke and Oke, 2021). Aspect ratios imply analyzing the optimal parametric setting by considering particular parts of parameters at the factor-level table definition. The aspect ratios are developed from direct parameters, which are single parameters without fractions. However, introducing other parameters as fractions of the rest parameters. Notwithstanding, Adegoke and Oke (2021) considered a mixture of direct parameters and aspect ratios to determine the optimal parametric settings of the process. In the study, three direct parameters, namely the percentage concentration of dispersed solids in the nanofluids, C, the cutting velocity, V, and the feed rate, F, are used.

Moreover, two aspect ratios, namely C/V and F/V are also developed with all the parameters used at the same time, subjected to three levels, for the determination of optimal parametric settings. However, the case discussed in the present study is different from the previously mentioned case in Adegoke and Oke (2021) along the following dimensions: (1) It analyses only aspect ratios in some cases while no such instance was reported previously in Adegoke and Oke (2021) but similar instance was reported in Adegoke et al. (2022) (2) The maximum power of either direct parameters alone, aspect ratios at one or a combination of direct parameters and aspect ratios for Adegoke and Oke (2021) and Adegoke et al. (2022) does not exceed unity while it reaches a power of two in the present study. (3) A power of his is commonly found in the dominator of aspect ratios used in the present work while it stands at unity for previously reported studies. (3) This work presents a comprehensive set of scenarios involving 83 options, but extremely few options are mentioned in the previous literature. Moreover, the power of direct parameters, which exceed one, shows more intense interactions of the parameters of the AA6062-T6 alloy in the friction stir welding process.

Adekoya et al., (2023) combined direct parameters and aspect ratios with eighteen scenarios that consider rotational analysis of parameters for the wear performance of nylon 6/Boron nitride (PA6/BN) composites. Although the study considers the third degree of parameters, at best the power of the three parameters was considered only once in each of the cases. However, a situation where at least the second degree of parameters occupation for two or more direct or aspect ratios at a time was not considered. Also, the friction stir welding application was ignored while the near phenomenon was treated.

Odudare et al. (2023) introduced the direct and aspect ratios to establish optimal parametric settings via on factor – level enroute Taguchi approach. As opposed to the present study, which considered four parameters as the direct parameters, this reviewed work is limited to only two parameters with the inclusion of squares and the reciprocals of squares as well as the cubes and their reciprocals as indirect factors. Thus, the synergic interaction between three and four parameters is omitted in the study. Oke and Adekoya (2022) considered only the aspect ratios in the evaluation of optimal parametric settings using the information on aspect ratios from the factor–level table. Compared with the present study, the maximum power of the aspect ratios is in the reviewed work. However, a higher power of two is considered in the present work.

2.2. Studies on aluminum alloys and friction stir welding

Orozco et al. (2013) made use of enhanced working parameters obtained from vibroacoustic signals brought out during the friction stir welding process to develop a mathematical model that tells the tensile strength of the welded joint for AA1050 aluminum alloy that has undergone friction stir welding. The input parameters for this model include travel speed, TP (tool profile), and RS (rotation speed) while the joint tensile strength and RMS (root mean square) of the vibroacoustic signal were the output as they are to be optimized. Liao and Daftardar (2013) made use of three mathematical models to study the behavior of AA.2195-T8 when friction stir welded. A thermal model was used to visualize the welding process of the material at given temperatures with its acting point on the material while a linear and non-linear model was used to analyze the relationship between this temperature and the three of the friction stir welding parameters. It was found that for optimum results to be achieved, there is a substantial dependence on the temperature, as we can get a higher bound of values for lower temperature while it gets more difficult to achieve for higher temperature values.

Rathinasuryan and Kumar (2020) look into enhancing the results of submerged friction stir welding of the 6061-T6 aluminum alloy with the use of RSM (response surface methodology), grey methods. They made use of GRA and ANOVA to determine the best combination of parametric values that gives the highest percentage elongation, and average hardness, and to verify the accuracy of the mathematical equations involved in the analysis, respectively. Jayabalakrishnan and Balasubramanian (2017) researched the improvement of microstructural characteristics and tensile strength of weld joints to improve the quality of weld finishing which can be achieved by choosing the best sets of values for step size (movement patterns of the tool), rotating speed and weaving rate among other friction stir welding parameters. The tensile strength of the welded joint of the AA6061-T6 aluminum alloy shows a high dependence on the weave pattern used as higher values were obtained for compact patterns with less spacing. Nandan et al. (2013) performed a simulated analysis on the three-dimensional viscoplastic flow of 304 austenitic stainless steel. The temperature field for this process was also visualized through simulation to help with calculations of properties that include viscosity (Non-Newtonian in this case) of the flowing metal. They discovered from the analysis that viscoplastic flow was concentrated at the tool surface and a minute section near the tool pin was occupied by transitive materials that became plastic. The magnitude of this section increases as we move closer to the tool shoulder. Singh and Hamilton (2013) utilized dynamic characterization with prediction models to study the connection between the weld energy and the dynamic interrogation variables to appraise the quality of the friction stir weld of 7136-T76 aluminum alloys. Employing a technique that causes no damage (No-destruction testing) to the material. The relationship between beam natural frequencies and weld energy was also looked into using theoretical models and dynamic tests that are experimentally scale-based.

Rodrigues et al. (2013) looked into the welding of non-heat-treatable AA5083 – H111 and heat-treatable AA608 – T6 aluminum alloys. The weldability of the two alloys was observed, focusing on the welding rotation and transverse speeds, axial loads, tilt angle, and dimensions of told that provide the best/highest welding speed. The experiment results for both materials are used to suggest a method that will effectively determine and stir the optimized values for each friction welding criterion considered. Pew et al. (2013) delved into the creation of a quantifiable mathematical model for both the input heat and weld power based on the friction stir welding condition variables. The heat input mathematical model demonstrated a relation between the spindle speed, feed rate, and heat input, as there are substantial decrements in the heat input when low spindle speed is used at low feed rates. Heat input is only affected by low federates as it is not affected much by the spindle speed at a high feed rate. Hashemzadeh et al. (2021) performed an experimental study coupled with numerical analysis of FSW (friction stir welding) of thick steel plates. The developed FEM for the welding heat source showed a satisfactory compatibility for the results gotten, both numerically and from the experiment. Sundqvist et al. (2017) used laser beams for preheating to reduce tool wear and created a mathematical model with the sole purpose of determining the forces acting on the tool during friction stir welding. It was found that preheating with a laser beam minimizes the forces experienced at the shoulder and pin of the friction stir welding tool and this is observed with a reduced generation of heat by the tool. Alam and Sinha (2021) conducted an experiment where AA2099 T8 Al-Li-based alloy is friction stir welded and the input parameters are studied to improve the tensile strength of the material while maintaining tool wear rate. Analytic models were designed for the tool wear rate, tensile models were designed for the tool wear rate, tensile strength, and durability of the composite with 91.04%, 95.94%, and 98.23% accuracy respectively. The optimum condition was given as 90mm per minute welding speed, 80% preheated welding temperature, 1500 rev per minute rotational speed of a cylindrical tool of the material, and 1.65% wear rate.

Gopi and Manonmani (2013) developed a mathematical model using the operating conditions and tool dimensions of FSW to determine the strength of the joining point of aluminum alloy 6082-T6 that was welded on both sides after the appropriate tests were carried out to check the tensile strength of the material, the operating conditions were related to the tensile strength and this was further analyze using ANOVA. Kundu and Singh (2017) researched how friction stir welding fares with joining sheets of aluminum 5083-H321 alloy using a stirring tool with the cylindrical profile. The implemented model gave a fundamental relationship of tool rotational speed, tilt angle of the tool, and other parameters with the joint's tensile strength and overall elongation of the combined sheets. The model showed that the predominant parameters were the tool rotating speed and tilt angle of the tool as they have the highest effects on the elongation and tensile strength at the joint, relative to other parameters.

Yunus and Alsoufi (2018) conducted research, focused on defining and describing how genetic programming can be used with friction welding to obtain the exact connection between the input parameters (axial load, spindle speed, plate thickness, tilt angle, and welding speed) and the output parameters (percentage elongation, joint strength, impact, tensile and yield strengths) to develop a model that can predict any outcome provided that values of the input parameters are given. The predicted results from the GP model showed an average 99.28% accuracy with experimentally obtained data, therefore this is a viable approach to predicting the behavior of several materials under FSW processes. Schmidt et al. (2023) conducted research into creating a systematic model for the generated heat during friction stir welding with assumptions on the type of contact between the welding tool and the weld piece. Results were obtained for the torque and plunging force experienced during the process and are further used to find out the type of contact between the welding tool and the weld piece. The sticking of the weld taken to the piece was observed at the tool surface as there was a direct relation of the generated heat to the plunging force. Savas et al. (2020) studied the ability of AA 6061 – T651 plates to transfer heat when undergoing friction stir welding for a specific period. An analysis was carried out using COMSOL code to stimulate the weld piece and the weld path as a rectangle-shaped aluminum plate and a straight line respectively. They found that for a spindle speed of 1500 KPM, there was a corresponding heat input of 130 Watts and this was obtained using a steady-state model.

Habba et al. (2018) studied the heat generation when the BT-FSW (bobbin tool friction stir welding) is used on AA1050 aluminum alloy by using a model that considers only the generated heat as its input parameter. Five different joints are welded, starting with a transverse speed of 200mm/min with increments of 200 for each consecutive joint till a maximum of 1000mm/min was performed per minute. It was found that as the spindle speed, shoulder diameter, pin size and coefficient of friction increase, the heat generated for the BT-FSW process also increases uniformly, while increments in the transverse speed of the bobbin tool result in reducing heat generation. Ghangas et al. (2022) researched the improvement of friction stir welding of AA5083 armor-grade aluminum alloy by obtaining the best working conditions for the process in terms of the process variables. It was realized that there was undesirable grain growth in the material region affected by heat when the heat input is on the high side, and of the lower heat input, the material does not melt and soften enough for appropriate welding of the material in the nugget region. Saeidi et al. (2015) observed the properties (microstructural and mechanical) of the welded joint of AA7075-T6 aluminum alloy and AA5083-H116 aluminum alloy at several speeds, both rotational and linear. The relation of tensile strength at the welded joint and the FSW process parameters is established by a synthesized mathematical model and enhanced by using GA (genetic algorithm friction stir welding. Results showed that the maximum attainable tensile strength was obtained at a spindle speed of 500 rad per minute and transverse speed of 50 mm per minute, with an error of 1% derivation from the experimental results.

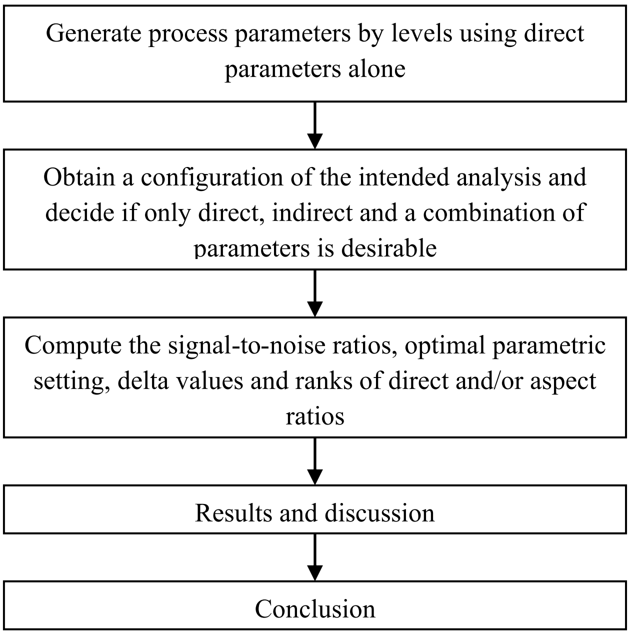

Figure 1. Research methodology flowchart

3. METHODOLOGY

The procedure followed to apply the aspect ratio-based Taguchi method, discussed in the present study, is as follows: The process was transformed by deploying experimental arrays with the help of orthogonal matrix L27. The source of an orthogonal array is the Minitab 18 version 2020. The parameters and levels were introduced to the platform of the Minitab, which transforms them into an orthogonal matrix. The outcome of this is generated experimental values which are used to compute the signal-to-noise ratios based on the larger-the-better criterion. However, it should be noted that from the factor-level table, the aspect ratios are first generated according to the formulation desired and an adjusted factor-level table is obtained. It is the new values that the signal-to-noise criterion of larger-the-better will introduce into the analysis. The direct parameters used in this work are labeled as TTA or A, TRS or B, and WS or C, indicating the tool tilt angle, tool rotational speed, and welding speed, respectively. Notice that the signal-to-noise ratio exhibits three types, but the nominal best criterion is not considered here as it does not apply to the case considered. Therefore Equations (1) and (2) are related to the criteria used in this work.

However, the higher-the-better criterion is used in the present study. The larger-the-better criterion is adopted because as the tensile strength of the AA6062-T6 alloy joint increases, it is beneficial to the system and desirable. Then, the values of the signal-to-noise ratios are obtained using the Microsoft Excel spreadsheet for analysis. Notice that the signal-to-noise ratios are considered for each experimental count to obtain the response table that produces the delta values and optimal parametric settings. Then the ranking of the parameters is done according to the dimension of the signal-to-noise ratios. In summary, Figure 1 shows the procedure used in this study.

In the Taguchi method, the two important formulae for signal-to-noise ratios are in Equations (1) and (2):

Smaller-the-better

$$SN = -10\log\frac{1}{n}\sum_{i=1}^{n} y_i^2 \tag{1}$$

Larger-the-better

$$SN = -10\log\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i^2} \tag{2}$$

where, Y is the response for the factor level, n is the number of responses in the factor level, $\sum$ is the summation, and is the iteration.

4. RESULTS AND DISCUSSIONS

When processing the AA6062-T6 alloy, the authors of the original work by Khan (2020) proposed three direct factors to be employed in determining the best parametric setting for the friction stir welding process. These are welding speed, tool rotation speed, and tool tilt angle, Khan, (2020), the best parametric setting was TTA3 TRS3 WS3, which is equivalent to 3° of TTA, 1400rpm of TRS, and 80mm/min of WS. The Taguchi method is obtained by designing experiments as a means of investigating the effect of the various parameters, Khan (Khan, 2020). These parameters and their levels are shown in Table 1.

Table 1. Process parameter by Khan (Khan, 2020)

Level	TTA (°)	TRS (rpm)	WS (mm/min)
1	2.0	700	40
2	2.5	1000	80
3	3.0	1400	110

The Taguchi for the aspect ratio is created by designing trials to study the impact of various characteristics; the first formulated aspect ratio is TTA.TRS and TTA/WS. Table 2 displays these variables and their corresponding levels. To create the L27 orthogonal array, these parameters were added to the Taguchi method. With the use of Minitab 18 version

2020, Table 3, the orthogonal array is produced using the two factors with three levels, and the parameters are configured. Table 3 shows how the operational parameters are computed using the array that was formed. To determine which parameter has a bigger impact on the welding process using the maxim "the larger the better." Thus, the values of (TTA/TRS) and (TTA/WS) based on the orthogonal array are substituted for the yi values in the computation of the signal-to-noise ratio, Equation (2). In Table 3, the signal-to-noise ratio has been determined and tabulated. Microsoft Excel program version 17 was utilized for analysis to streamline the process. This process was used to determine the signal-to-noise ratio of all 83 aspect ratio formulations as shown in Table 3.

Table 2. Response table

Level	A/B	A/C
1	0.002857	0.050000
2	0.002500	0.031250
3	0.002143	0.027273

Optimal parametric setting: $(A/B)_1(A/C)_1$

Table 3. Signal-to-noise ratios

Sr. No.	SN	Sr. No.	SN
1	-21.2352	15	-25.3041
2	-21.2352	16	-26.4787
3	-21.2352	17	-26.4787
4	-25.2956	18	-26.4787
5	-25.2956	19	-21.2414
6	-25.2956	20	-21.2414
7	-26.4677	21	-21.2414
8	-26.4677	22	-25.3114
9	-26.4677	23	-25.3114
10	-21.2385	24	-25.3114
11	-21.2385	25	-26.4883
12	-21.2385	26	-26.4883
13	-25.3041	27	-26.4883
14	-25.3041		

However, to benchmark these results against those provided by the aspect-based Taguchi method in formation one, formation one is first defined as the joint parameters of TTA/TRS and TTA/WS. The optimal parametric setting that this yielded is (TTA/TRS)1 and (TTA/WS)1. This is interpreted as 0.002857(° /rpm) of TTA/TRS AND 0.05(°/mm min^{-1}). However, there is no basis for comparing two entire parameters with different units formed in the formation of the aspect ratio. But in the original result by Khan (2020) three direct factors are used. This means a new way of evaluating the optimal parametric setting using the aspect ratio of Taguchi has been proposed. By following this thought pattern for the rest of the 82 cases, Table 1 is the summary of the result. Table 1 summarizes the results for the 82 formulation. When you have the L27 orthogonal array with its parameters and levels, you can square this parameter to get the values you want. Then, you can add each row of parameters and divide each sum by one to get the values (Fig. 1). The result is then multiplied by (1/n), where n is the total number of elements, which is 3. The signal-to-noise ratio (SNR), shown in Table 2, is then calculated by taking the logarithm of this number and multiplying it by (-10) To determine each parameter's impact on the welding process, the average signal-to-noise (SN) value is calculated for each factor and level after the SNR for the experiment has been determined, as shown in Table 4.

Table 4. Signal-to-noise response table – Taguchi Aspect ratios

Level	A/B	A/C
1	-24.33284815	-21.23839706
2	-24.34045380	-25.30371008
3	-24.34705848	-26.47825330
Delta	0.014210330	5.239856238
Ranking	2nd	1st

Optimal parametric setting: $(A/B)_1(A/C)_1$

The maximum value in the parameter less the minimum value along the column is used to determine the delta value of the SNR for each parameter. In Table 4, the range for the parameter (TTA/TRS) is derived. The objective function to be considered is likewise decided using the delta value. The more significant a parameter's ranking value, the more influence it has on the process. According to Table 4, (TTA/WS) comes in first place with a score of 5.239856238, followed by 0.0534. As can be observed, (TTA/WS) has a more significant impact on the welding process than (TTA/WS).

For the optimal parametric setting, Level 1 has (-24.33284815) as the highest value for the parameter (TTA/TRS) among the three levels and for TTA/WS level 1 also has the highest value among the three levels of (-21.23839706). The optimal parametric setting is A/B1A/C1.

In this article, a wide range of scenarios (i.e. 83 cases) was presented. In each case, the optimal parametric setting is obtained as a function of the aspect ratios or a combination of direct parameters and aspect ratios. The optimal parametric setting presented is completely different in each case from the traditional results offered by the direct parameters alone. This then poses a challenge on how to compare them and offer conclusions for further decision making. It is noticed that the optimal parametric setting may be single or multiple depending on the number of parameters considered in the analysis. It was found that a factor-level table of as simple as two aspect ratios and three levels may produce only one optimal parametric setting. For instance, in Formulation 1, where the aspect ratios considered are only two, namely, TTA/TRS and TTA/WS, the optimal parametric setting is single, namely $(A/B)_1(A/C)_1$ or more specifically stated as $(TTA/TRS)_1(TTA/WS)_1$.

Moreover, considering more complicated interactions of the parameters, different results may be obtained. As an instance, consider scenario 83 where six aspect ratios are analysed. Here, the results show three optimal variants, notably BC/A_3 AC/B_1, AB/C_1 A^2_1, B^2_3 C^2_3 as the first setting. The second parametric setting is BC/A_3 AC/B_2 A^2_2 $B^2$3 while the third optimal parametric setting is BC/A_3 AC/B_3 AB/C_3 A^2_3 B^2_3 C^2_3. This implies that any of these three options of the optimal parametric setting for scenario 84 could be used for further decision-making (Table 5).

Table 5. Scenarios in the optimal parametric setting

<table>
<tr><th>Formulation</th><th>Optimal parametric setting and interpretation</th></tr>
<tr><td>Formulation1
<table>
<tr><td>Level</td><td>TTA/TRS</td><td>TTA/WS</td></tr>
<tr><td>1</td><td>0.002857</td><td>0.05</td></tr>
<tr><td>2</td><td>0.0025</td><td>0.03125</td></tr>
<tr><td>3</td><td>0.002143</td><td>0.027273</td></tr>
</table>
Optimal parametric setting: $(A/B)_1(A/C)_1$</td><td>joint TTA/TRS and TTA/WS yield (TTA/TRS)1 and (TTA/WS)1 to obtain 0.002857 (rpm/ ^{0}) and 0.05 (rpm(min)/mm).</td></tr>
<tr><td>Formulation 2
<table>
<tr><td>Level</td><td>B/A</td><td>B/C</td></tr>
<tr><td>1</td><td>350</td><td>17.5</td></tr>
<tr><td>2</td><td>400</td><td>12.5</td></tr>
<tr><td>3</td><td>466.6667</td><td>12.72727</td></tr>
</table>
Optimal parametric setting: $(B/A)_3(B/C)_3$</td><td>joint TRS/TTA and TRS/WS yield (TRS/TTA)3 and (TRS/WS)3 to obtain 466.6667 (rpm/^{0}) and 12.72727 (rpm (min)/min)</td></tr>
<tr><td>Formulation 3
<table>
<tr><td>Level</td><td>C/A</td><td>C/B</td></tr>
<tr><td>1</td><td>20</td><td>0.057143</td></tr>
<tr><td>2</td><td>32</td><td>0.08</td></tr>
<tr><td>3</td><td>36.66667</td><td>0.078571</td></tr>
</table>
Optimal parametric setting: $A/B_3A/C_2$</td><td>joint WS/TTA and WS/TRS yield (WS/TTA)3 and (WS/TRS)2 to obtain 36.66667 (mm/ ^{0}.min), 0.08 (mm/min.rpm).</td></tr>
<tr><td>Formulation 4
<table>
<tr><td>Level</td><td>A/B</td><td>A/C</td><td>A</td><td>B</td><td>C</td></tr>
<tr><td>1</td><td>0.002857143</td><td>0.05</td><td>2</td><td>700</td><td>40</td></tr>
<tr><td>2</td><td>0.0025</td><td>0.03125</td><td>2.5</td><td>1000</td><td>80</td></tr>
<tr><td>3</td><td>0.002142857</td><td>0.027273</td><td>3</td><td>1400</td><td>110</td></tr>
</table>
Optimal parametric setting: $A/B_2A/C_2A_3B_3C_3$</td><td>joint TTA/TRS, TTA/WS, TTA, TRS, and WS yield (TTA/TRS)2 (TTA/WS)2 (TTA)3 (TRS)3 (WS)3 to obtain 0.0025(^{0}/rpm), 0.03125 (^{0}min/mm), 3^0, 140 rpm, 110 mm/min.</td></tr>
<tr><td>Formulation 5
<table>
<tr><td>Level</td><td>B/A</td><td>B/C</td><td>A</td><td>B</td><td>C</td></tr>
<tr><td>1</td><td>350</td><td>17.5</td><td>2</td><td>700</td><td>40</td></tr>
<tr><td>2</td><td>400</td><td>12.5</td><td>2.5</td><td>1000</td><td>80</td></tr>
<tr><td>3</td><td>466.6667</td><td>12.7273</td><td>3</td><td>1400</td><td>110</td></tr>
</table>
Optimal parametric setting: $A/B_3A/C_3A_2B_3C_3$</td><td>joint TRS/TTA, TRS/WS, TTA, TRS, and WS yield (TRS/TTA)3 (TRS/WS)3 (TTA)2 (TRS)3 (WS)3 to obtain 466.67(^{0}/rpm), 12.73 (^{0}min/mm), 2.5^0, 1400 rpm, 110 mm/min.</td></tr>
<tr><td>Formulation 6
<table>
<tr><td>Level</td><td>C/A</td><td>C/B</td><td>A</td><td>B</td><td>C</td></tr>
<tr><td>1</td><td>20</td><td>0.0571</td><td>2</td><td>700</td><td>40</td></tr>
<tr><td>2</td><td>32</td><td>0.08</td><td>2.5</td><td>1000</td><td>80</td></tr>
<tr><td>3</td><td>36.6667</td><td>0.0786</td><td>3</td><td>1400</td><td>110</td></tr>
</table>
Optimal parametric setting: $A/B_3A/C_3A_2B_3C_3$</td><td>joint WS/TTA, WS/TRS, TTA, TRS, and WS yield (WS/TTA)3 (WS/TRS)3 (TTA)2 (TRS)3 (WS)3 to obtain 36.67 (^{0}/rpm), 0.0786 (^{0}min/mm), 2.5^0, 1400 rpm, 110 mm/min.</td></tr>
<tr><td>Formula 7
<table>
<tr><td>Level</td><td>A^2</td><td>B^2</td><td>C^2</td></tr>
<tr><td>1</td><td>4</td><td>490000</td><td>1600</td></tr>
<tr><td>2</td><td>6.25</td><td>1000000</td><td>6400</td></tr>
<tr><td>3</td><td>9</td><td>1960000</td><td>12100</td></tr>
</table>
Optimal parametric setting: $A^23B^23C^23$</td><td>joint TTA^2, TRS^2 and WS^2 yield (TTA^2)3 (TRS^2)3 (WS^2)2 to obtain 9 (^{0}/rpm), 1960000 (^{0}.min/mm), 2.5^0, 12100 mm/min.</td></tr>
<tr><td>Formula 8
<table>
<tr><td>A^2</td><td>B^2</td><td>C^2</td><td>A</td><td>B</td><td>C</td></tr>
<tr><td>4</td><td>490000</td><td>1600</td><td>2</td><td>700</td><td>40</td></tr>
<tr><td>6.25</td><td>1000000</td><td>6400</td><td>2.5</td><td>1000</td><td>80</td></tr>
<tr><td>9</td><td>1960000</td><td>12100</td><td>3</td><td>1400</td><td>110</td></tr>
</table>
Optimal parametric setting: $A^23B^23C^23A2B3C1$</td><td>joint TTA^2, TRS^2, WS^2,TTA, TRS and WS yield (TTA^2)3 (TRS^2)3 (WS^2)3 (TTA)2, (TRS)3, (WS)1 to obtain 9(^{0}/rpm), 1960000(^{0}min/mm), 12100mm/min, 2,5(^{0}) 1400rpm,40mm/min.</td></tr>
<tr><td>Formula 9</td><td>joint TTA, TTA^2/TRS and TTA^2/WS yield (TTA)3 (TTA^2/TRS)1 (TTA)^2/WS)1 to obtain 3(^{0}/rpm), 0.005714(^{0}min/mm), 2.5^0, 1400rpm, 0.1mm/min.</td></tr>
<tr><td>Formula 10</td><td>Joint TTA, TTA^2/TRS, TTA^2/WS, TTA, TRS, WS Yield (TTA) (TTA^2/WS) (TTA^2/WS) (TTA) (TRS) (WS) to obtain 0.006429(^{0}), 0.081818(^{0}/rpm), 3(^{0}min/mm), 3(^{0}), 1400(rpm), 110(mm/min).</td></tr>
</table>

<table>
<tr><th>Formulation</th><th>Optimal parametric setting and interpretation</th></tr>
<tr><td>Formula 11</td><td>joint TRS^2/TTA, TRS^2/TRS and TRS^2/WS yield (TRS^2/TTA)3 (TRS^2/TRS)1 (TRS^2/WS)1 to obtain 653333.33 (rpm/^{0}), 1400 (rpm), 17818.182(rpm. min/mm).</td></tr>
<tr><td>Formula 12</td><td>Joint TRS^2/TTA, TRS^2/TRS, TRS^2/WS, TTA, TRS, WS Yield (TRS^2/TTA)3 (TRS^2/TRS)3 (TRS^2/WS)3 (TTA)3 (TRS)3 (WS)3 to obtain 653333.3 (rpm/^{0}), 1400 (rpm), 17818.18 (rpm.min/mm), 3(^{0}), 1400(rpm), 110(mm/min).</td></tr>
<tr><td>Formula 13</td><td>joint WS^2/TTA, WS^2/TRS and WS^2/WS yield (WS^2/TTA)3 (WS^2/TRS)3 (WS^2/WS)3 to obtain 4033.333 (mm/min /^{0}), 8.642857 (mm/min.rpm), 110 (min/mm).</td></tr>
<tr><td>Formula 14</td><td>joint WS^2/TTA, WS^2/TRS, WS^2/WS, TTA, TRS, and WS yield (WS^2/TTA)3 (WS^2/TRS)3 (WS^2/WS)3 (TTA)3 (TRS)3 (WS)3 to obtain 4033.333 (mm/min /^{0}), 8.642857 (mm/min.rpm), 110 (min/mm) 3(^{0}), 1400 (rpm), 110 (mm/min).</td></tr>
<tr><td>Formula 15</td><td>joint TTA^2/TRS and TTA^2/WS yield (TTA^2/TRS)3 and (TTA^2/WS)1 to obtain 0.006428571 (^{0}/rpm) and 0.1 (^{0}/rpm).</td></tr>
<tr><td>Formula 16</td><td>joint TTA^2/TRS, TTA^2/WS, (TTA)3 (TRS)3 and (WS)3 yield (TTA^2/TRS)3 (TTA^2/WS)3 (TTA)2 (TRS)3 (WS)3 to obtain 4.59184E-06 (^{0}/rpm), 0.000744 (^{0}/rpm) 2.5(^{0}), 1400(rpm), 110(mm/min).
A/B3A/C3A2B3C3</td></tr>
<tr><td>Formula 17</td><td>joint TTA^2/TRS, TTA^2/WS, (TTA)3 (TRS)3 and (WS)3 yield (TTA^2/TRS)1 (TTA^2/WS)1 (TTA)1 (TRS)3 (WS)3 to obtain 8.16327E-06 (^{0}/rpm), 0.0025 (^{0}/rpm) 4(^{0}), 1960000 (rpm), 12100 (mm/min).
joint TTA^2/TRS, TTA^2/WS, (TTA)3 (TRS)3 and (WS)3 yield (TTA^2/TRS)2 (TTA^2/WS)2 (TTA)2 (TRS)3 (WS)3 to obtain 0.00000625 (^{0}/rpm), 0.000977 (^{0}/rpm) 6.25 (^{0}), 1960000 (rpm), 12100 (mm/min).
joint TTA^2/TRS, TTA^2/WS, (TTA)3 (TRS)3 and (WS)3 yield (TTA^2/TRS)3 (TTA^2/WS)3 (TTA)3 (TRS)3 (WS)3 to obtain 4.59184E-06 (^{0}/rpm), 0.000744 (^{0}/rpm) 9(^{0}), 1960000 (rpm), 12100 (mm/min).
$A^2/B^2$1 $A^2/C^2$1 $A^2$1 $B^2$3 $C^2$3
$A^2/B^2$2 $A^2/C^2$2 $A^2$2 $B^2$3 $C^2$3
$A^2/B^2$3 $A^2/C^2$3 $A^2$3 $B^2$3 $C^2$3</td></tr>
<tr><td>Formula 18
<table>
<tr><th>Level</th><th>A^2/B^2</th><th>A^2/C^2</th><th>A^2</th><th>B^2</th><th>C^2</th></tr>
<tr><td>1</td><td>8.1633E-06</td><td>0.0025</td><td>4</td><td>490000</td><td>1600</td></tr>
<tr><td>2</td><td>0.00000625</td><td>0.0010</td><td>6.25</td><td>1000000</td><td>6400</td></tr>
<tr><td>3</td><td>4.5918E-06</td><td>0.0007</td><td>9</td><td>1960000</td><td>12100</td></tr>
</table>
Optimal parametric settings: $A^2/B^2$1 $A^2/C^2$1 $A^2$1 $B^2$3 $C^2$3; $A^2/B^2$2 $A^2/C^2$2 $A^2$2 $B^2$3 $C^2$3; $A^2/B^2$3 $A^2/C^2$3 $A^2$3 $B^2$3 $C^2$3</td><td>Joint TTA^2/TRS^2, TTA^2/WS^2, (TTA^2)3 (TRS^2)3 and (WS^2)3 yield (TTA^2/TRS^2)1 (TTA^2/WS^2)1 (TTA^2)1 (TRS^2)3 (WS^2)3 to obtain 8.16327E-06 (^{0}/rpm), 0.0025 (^{0}/rpm) 4(^{0}), 1960000 (rpm), 12100 (mm/min).
joint TTA^2/TRS^2, TTA^2/WS^2, (TTA^2)3 (TRS^2)3, and (WS^2)3 yield (TTA^2/TRS^2)2 (TTA^2/WS^2)2 (TTA^2)2 (TRS^2)3 (WS^2)3 to obtain 0.00000625 (^{0}/rpm) 0.000977 (^{0}/rpm) 2.5(^{0}), 1960000 (rpm), 12100 (mm/min).</td></tr>
</table>

<table>
<tr><th>Formulation</th><th>Optimal parametric setting and interpretation</th></tr>
<tr><td></td><td>joint TTA^2/TRS^2, TTA^2/WS^2, (TTA^2)3 (TRS^2)3 and (WS^2)3 yield (TTA^2/TRS^2)3 (TTA^2/WS^2)3 (TTA^2)3 (TRS^2)3 (WS^2)3 to obtain 4.59184E-06 (^{0}/rpm), 0.000744 (^{0}/rpm) 9(^{0}), 1960000 (rpm), 12100 (mm/min).</td></tr>
<tr><td>Formula 19
<table>
<tr><td>Level</td><td>B^2/A^2</td><td>B^2/C^2</td></tr>
<tr><td>1</td><td>122500</td><td>306.25</td></tr>
<tr><td>2</td><td>160000</td><td>156.25</td></tr>
<tr><td>3</td><td>217777.778</td><td>161.9835</td></tr>
</table>
Optimal parametric setting: $B^2/A^2$3$B^2/C^2$1</td><td>Joint TRS^2/TTA^2, TRS^2/WS^2 Yield (TRS^2/TRS^2)3 (TRS^2/WS^2)1 to obtain 217777.778 (rpm/^{0}), 306.25 (rpm. min/mm).</td></tr>
<tr><td>Formula 20
<table>
<tr><td>Level</td><td>B^2/A^2</td><td>B^2/C^2</td><td>A</td><td>B</td><td>C</td></tr>
<tr><td>1</td><td>122500</td><td>306.25</td><td>2</td><td>700</td><td>40</td></tr>
<tr><td>2</td><td>160000</td><td>156.25</td><td>2.5</td><td>1000</td><td>80</td></tr>
<tr><td>3</td><td>217777.8</td><td>161.9835</td><td>3</td><td>1400</td><td>110</td></tr>
</table>
Optimal parametric setting: $B^2/A^2$3 $B^2/C^2$3 A3 B3 C3</td><td>Joint TRS^2/TTA^2, TRS^2/WS^2 (TTA) (TRS) (WS) Yield (TRS^2/TRS^2)3 (TRS^2/WS^2)3 (TTA)3 (TRS)3 (WS)3 to obtain 217777.778 (rpm/^{0}), 161.9835 (rpm.min/mm), 3(^{0}), 1400 (rpm), 110 (mm/min).</td></tr>
<tr><td>Formula 21
<table>
<tr><td>Level</td><td>B^2/A^2</td><td>B^2/C^2</td><td>A^2</td><td>B^2</td><td>C^2</td></tr>
<tr><td>1</td><td>122500</td><td>306.25</td><td>4</td><td>490000</td><td>1600</td></tr>
<tr><td>2</td><td>160000</td><td>156.25</td><td>6.25</td><td>1000000</td><td>6400</td></tr>
<tr><td>3</td><td>217777.8</td><td>161.9835</td><td>9</td><td>1960000</td><td>12100</td></tr>
</table>
Optimal parametric setting: $B^2/A^2$3 $B^2/C^2$3 $A^2$2 $B^2$3 $C^2$3</td><td>Joint TRS^2/TTA^2, TRS^2/WS^2 (TTA^2) (TRS^2) (WS^2) Yield (TRS^2/TRS^2)3 (TRS^2/WS^2)3 (TTA^2)2 (TRS^2)3 (WS^2)3 to obtain 217777.778 (rpm/^{0}), 161.9835 (rpm.min/mm), 6.25(^{0}), 1960000 (rpm), 12100 (mm/min).</td></tr>
<tr><td>Formula 22
<table>
<tr><td>Level</td><td>C^2/A^2</td><td>C^2/B^2</td></tr>
<tr><td>1</td><td>400</td><td>0.0033</td></tr>
<tr><td>2</td><td>1024</td><td>0.0064</td></tr>
<tr><td>3</td><td>1344.444</td><td>0.0062</td></tr>
</table>
Optimal parametric setting: $C^2/A^2$3$C^2/B^2$3</td><td>Joint WS^2/TTA^2, WS^2/TRS^2 Yield (WS^2/TTA^2)3 (WS^2/TRS^2)3 to obtain 1344.444 (mm/min /^{0}), 0.006173 (mm/min.rpm).</td></tr>
<tr><td>Formula 23
<table>
<tr><td>Level</td><td>C^2/A^2</td><td>C^2/B^2</td><td>A</td><td>B</td><td>C</td></tr>
<tr><td>1</td><td>400</td><td>0.0033</td><td>2</td><td>700</td><td>40</td></tr>
<tr><td>2</td><td>1024</td><td>0.0064</td><td>2.5</td><td>1000</td><td>80</td></tr>
<tr><td>3</td><td>1344.444</td><td>0.0062</td><td>3</td><td>1400</td><td>110</td></tr>
</table>
Optimal parametric setting: $C^2/A^2$3$C^2/B^2$3A3B3C3</td><td>Joint WS^2/TTA^2, WS^2/TRS^2, (TTA) (TRS) (WS) Yield (WS^2/TTA^2)3 (WS^2/TRS^2)3 (TTA)3 (TRS)3 (WS)3 to obtain 1344.444 (mm/min /^{0}), 0.006173 (mm/min.rpm), 3(^{0}), 1400 (rpm), 110 (mm/min).</td></tr>
<tr><td>Formula 24
<table>
<tr><td>Level</td><td>C^2/A^2</td><td>C^2/B^2</td><td>A</td><td>B</td><td>C</td></tr>
<tr><td>1</td><td>400</td><td>0.0033</td><td>4</td><td>490000</td><td>1600</td></tr>
<tr><td>2</td><td>1024</td><td>0.0064</td><td>6.25</td><td>1000000</td><td>6400</td></tr>
<tr><td>3</td><td>1344.444</td><td>0.0062</td><td>9</td><td>1960000</td><td>12100</td></tr>
</table>
Optimal parametric setting: $C^2/A^2$3$C^2/B^2$3$A^2$2$B^2$3C^2</td><td>Joint WS^2/TTA^2, WS^2/TRS^2, (TTA^2) (TRS^2) (WS^2) Yield (WS^2/TTA^2)3 (WS^2/TRS^2)3 (TTA^2)3 (TRS^2)3 (WS^2)3 to obtain 1344.444 (mm/min /^{0}), 0.006173 (mm/min.rpm), 9(^{0}), 1960000 (rpm), 12100 (mm/min).</td></tr>
<tr><td>Formula 25
<table>
<tr><td>Level</td><td>A/B</td><td>A/C</td><td>A^2</td><td>B^2</td><td>C^2</td></tr>
<tr><td>1</td><td>0.0029</td><td>0.05</td><td>4</td><td>490000</td><td>1600</td></tr>
<tr><td>2</td><td>0.0025</td><td>0.03125</td><td>6.25</td><td>1000000</td><td>6400</td></tr>
<tr><td>3</td><td>0.0021</td><td>0.0273</td><td>9</td><td>1960000</td><td>12100</td></tr>
</table>
Optimal parametric setting: A/B1 A/C1 $A^2$1 $B^2$3 $C^2$3; A/B2 A/C2 $A^2$2 $B^2$3 $C^2$3; A/B3 A/C3 $A^2$3 $B^2$3 $C^2$3</td><td>joint TTA/TRS, TTA/WS, (TTA)^2 (TRS)^2 and (WS)^2 yield (TTA/TRS)1 (TTA/WS)1 (TTA^2)1 (TRS^2)3 (WS^2)3 to obtain 0.002857 (^{0}/rpm), 0.05 (^{0}min./mm) 2.5(^{0}), 1960000 (rpm), 12100 (mm/min).
joint TTA/TRS, TTA/WS, (TTA)^2 (TRS)^2 and (WS)^2 yield (TTA/TRS)2 (TTA/WS)2 (TTA^2)2 (TRS^2)3 (WS^2)3 to obtain 0.0025 (^{0}/rpm), 0.03125 (^{0}min./mm) 6.25 (^{0}), 1960000 (rpm), 12100 (mm/min).
joint TTA/TRS, TTA/WS, (TTA)^2 (TRS)^2 and (WS)^2 yield (TTA/TRS)3 (TTA/WS)3 (TTA^2)3 (TRS^2)3 (WS^2)3 to obtain 0.002143 (^{0}/rpm), 0.05 (^{0}min./mm) 2.5(^{0}), 1960000 (rpm), 12100 (mm/min).</td></tr>
</table>

<table>
<tr><th>Formulation</th><th>Optimal parametric setting and interpretation</th></tr>
<tr><td>Formula 26
<table>
<tr><td>Level</td><td>B/A</td><td>B/C</td><td>A^2</td><td>B^2</td><td>C^2</td></tr>
<tr><td>1</td><td>350</td><td>17.5</td><td>4</td><td>490000</td><td>1600</td></tr>
<tr><td>2</td><td>400</td><td>12.5</td><td>6.25</td><td>1000000</td><td>6400</td></tr>
<tr><td>3</td><td>466.6667</td><td>12.7273</td><td>9</td><td>1960000</td><td>12100</td></tr>
</table>
Optimal parametric setting: B/A3B/A3$A^2$2$B^2$3$C^2$2</td><td>joint TRS/TTA, TRS/WS, (TTA^2), (TRS^2) and (WS^2) yield (TRS/TTA)3 (TRS/WS)3 (TTA^2)2 (TRS^2)3 (WS^2)2 to obtain 466.67(^{0}/rpm), 12.73(^{0}min/mm), 4(^{0}), 1960000(rpm), 6400 (mm/min).</td></tr>
<tr><td>Formula 27
<table>
<tr><td>Level</td><td>A^2</td><td>B^2</td><td>C^2</td></tr>
<tr><td>1</td><td>4</td><td>490000</td><td>1600</td></tr>
<tr><td>2</td><td>6.25</td><td>1000000</td><td>6400</td></tr>
<tr><td>3</td><td>9</td><td>1960000</td><td>12100</td></tr>
</table>
Optimal parametric setting: $A^2$3$B^2$3$C^2$1</td><td>joint TTA^2, TRS^2 and WS^2 yield (TTA^2)3 (TRS^2)3 (WS^2)1 to obtain 9(^{0}/rpm), 1960000(^{0}min/mm), 2.5(^{0}), 1600(mm/min).</td></tr>
<tr><td>Formula 28
<table>
<tr><td>Level</td><td>AB/A</td><td>AB/B</td><td>AB/C</td></tr>
<tr><td>1</td><td>700</td><td>2</td><td>35</td></tr>
<tr><td>2</td><td>1000</td><td>2.5</td><td>31.25</td></tr>
<tr><td>3</td><td>1400</td><td>3</td><td>38.1818</td></tr>
</table>
Optimal parametric setting: AB/A3AB/B3AB/C3</td><td>joint TTA.TRS/TTA, TTA.TRS/TRS and TTA.TRS/WS yield TTA.TRS/TTA, TTA.TRS/TRS TTA.TRS/WS to obtain 9(rpm), 1960000(^{0}), 2.5(^{0})rpm.min/mm).</td></tr>
<tr><td>Formula 29
<table>
<tr><td>Level</td><td>AB/A</td><td>AB/B</td><td>AB/C</td><td>A</td><td>B</td><td>C</td></tr>
<tr><td>1</td><td>700</td><td>2</td><td>35</td><td>2</td><td>700</td><td>40</td></tr>
<tr><td>2</td><td>1000</td><td>2.5</td><td>31.25</td><td>2.5</td><td>1000</td><td>80</td></tr>
<tr><td>3</td><td>1400</td><td>3</td><td>38.18182</td><td>3</td><td>1400</td><td>110</td></tr>
</table>
Optimal parametric setting: AB/A3AB/B3AB/C3A3B3C3</td><td>joint (TTA.TRS/TTA), (TTA.TRS/TRS), (TTA.TRS/WS), (TTA) (TRS) and (WS) yield (TTA.TRS/TTA)3, (TTA.TRS/TRS)3, (TTA.TRS/WS)3, (TTA)3 (TRS)3 (WS)3 to obtain 1400 (rpm), 3(^{0}), 2(^{0})rpm.min/mm, 3(^{0}), 1400 (rpm), 110 (mm/min).</td></tr>
<tr><td>Formula 30
<table>
<tr><td>Level</td><td>AB/A</td><td>AB/B</td><td>AB/C</td><td>A^2</td><td>B^2</td><td>C^2</td></tr>
<tr><td>1</td><td>700</td><td>2</td><td>35</td><td>4</td><td>490000</td><td>1600</td></tr>
<tr><td>2</td><td>1000</td><td>2.5</td><td>31.25</td><td>6.25</td><td>1000000</td><td>6400</td></tr>
<tr><td>3</td><td>1400</td><td>3</td><td>38.1818</td><td>9</td><td>1960000</td><td>12100</td></tr>
</table>
Optimal parametric setting: AB/A3AB/B3AB/C3$A^2$3$B^2$3$C^2$3</td><td>joint (TTA.TRS/TTA), (TTA.TRS/TRS), (TTA.TRS/WS), (TTA^2) (TRS^2) and (WS^2) yield (TTA.TRS/TTA)3, (TTA.TRS/TRS)3, (TTA.TRS/WS)3, (TTA^2)3 (TRS^2)3 (WS^2)3 to obtain 1400 (rpm), 3(^{0}), 2(^{0})rpm.min/mm, 9(^{0}), 0 (rpm), 12100(mm/min).</td></tr>
<tr><td>Formula 31
<table>
<tr><td>Level</td><td>AB/A</td><td>AB/B</td><td>AB/C</td></tr>
<tr><td>1</td><td>700</td><td>2</td><td>35</td></tr>
<tr><td>2</td><td>1000</td><td>2.5</td><td>31.25</td></tr>
<tr><td>3</td><td>1400</td><td>3</td><td>38.1818</td></tr>
</table>
Optimal parametric setting: CA/A3CA/B3CA/C3</td><td>joint (WS.TTA/TTA), (WS.TTA/TRS), and (WS.TTA/WS) yield (WS.TTA/TTA)3, (WS.TTA/TRS)3, (WS.TTA/WS)3 to obtain 110 (rpm), 0.235714 (^{0}), 3(^{0}.rpm.min/mm).</td></tr>
<tr><td>Formula 32
<table>
<tr><td>Level</td><td>CA/A</td><td>CA/B</td><td>CA/C</td><td>A</td><td>B</td><td>C</td></tr>
<tr><td>1</td><td>40</td><td>0.1143</td><td>2</td><td>2</td><td>700</td><td>40</td></tr>
<tr><td>2</td><td>80</td><td>0.2</td><td>2.5</td><td>2.5</td><td>1000</td><td>80</td></tr>
<tr><td>3</td><td>110</td><td>0.2357</td><td>3</td><td>3</td><td>1400</td><td>110</td></tr>
</table>
Optimal parametric setting: CA/A3CA/B3CA/C3A3B3C3</td><td>joint (WS.TTA/TTA), (WS.TTA/TRS), (WS.TTA/WS), (TTA) (TRS) and (WS) yield (WS.TTA/TTA)3, (WS.TTA/TRS)3, (WS.TTA/WS)3, (TTA)3 (TRS)3 (WS)3 to obtain 110 (rpm), 6(^{0}), 3 (^{0}.rpm.min/mm), 3(^{0}), 1400 (rpm), 110 (mm/min).</td></tr>
<tr><td>Formula 33
<table>
<tr><td>Level</td><td>CA/A</td><td>CA/B</td><td>CA/C</td><td>A^2</td><td>B^2</td><td>C^2</td></tr>
<tr><td>1</td><td>40</td><td>0.1143</td><td>2</td><td>4</td><td>490000</td><td>1600</td></tr>
<tr><td>2</td><td>80</td><td>0.2</td><td>2.5</td><td>6.25</td><td>1000000</td><td>6400</td></tr>
<tr><td>3</td><td>110</td><td>0.2357</td><td>3</td><td>9</td><td>1960000</td><td>12100</td></tr>
</table>
Optimal parametric setting: CA/A3CA/B3CA/C3$A^2$3$B^2$3$C^2$3</td><td>joint (WS.TTA/TTA), (WS.TTA/TRS), (WS.TTA/WS), (TTA^2) (TRS^2) (WS^2) yield (WS.TTA/TTA)3, (WS.TTA/TRS)3, (WS.TTA/WS)3, (TTA^2)3 (TRS^2)3 (WS^2)3 to obtain 110 (rpm), 4(^{0}), 3(^{0}.rpm.min/mm), 9(^{0}), 0 (rpm), 12100 (mm/min).</td></tr>
<tr><td>Formula 34
<table>
<tr><td>Level</td><td>A/A^2</td><td>A/B^2</td><td>A/C^2</td></tr>
<tr><td>1</td><td>0.5</td><td>4.08163E-06</td><td>0.00125</td></tr>
<tr><td>2</td><td>0.4</td><td>0.0000025</td><td>0.0004</td></tr>
<tr><td>3</td><td>0.3333</td><td>1.53061E-06</td><td>0.0002</td></tr>
</table>
Optimal parametric setting: $(A/A^2)_1(A/B^2)_2(A/C^2)_3$</td><td>joint (1/TTA^2), (TTA/TRS^2), and (TTA/WS^2) yield (1/TTA^2)1, (TTA/TRS^2)2, (TTA/WS^2)1 to obtain 110 (1/^{0}), 0.235714 (^{0}/rpm), 3 (^{0}.rpm.(min/mm)^2).</td></tr>
</table>

<table>
<tr><th>Formulation</th><th>Optimal parametric setting and interpretation</th></tr>
<tr><td>Formula 35
<table>
<tr><td>Level</td><td>A/A^2</td><td>A/B^2</td><td>A/C^2</td><td>A</td><td>B</td><td>C</td></tr>
<tr><td>1</td><td>0.5</td><td>4.0816E-06</td><td>0.00125</td><td>2</td><td>700</td><td>40</td></tr>
<tr><td>2</td><td>0.4</td><td>0.000003</td><td>0.0004</td><td>2.5</td><td>1000</td><td>80</td></tr>
<tr><td>3</td><td>0.3333</td><td>1.5306E-06</td><td>0.0002</td><td>3</td><td>1400</td><td>110</td></tr>
</table>
Optimal parametric setting: $A/A^2 3 A/B^2 3 A/C^2 3A3B3C3$</td>
<td>joint (1/TTA^2), (TTA/TRS^2), (TTA/WS^2), (TTA) (TRS) and (WS) yield (1/TTA^2)3, (TTA/TRS^2)3, (TTA/WS^2)3 (TTA)3 (TRS)3 (WS)3 to obtain 0.333333 (1/^{0}), 1.5306E-06 (^{0}/rpm), 0.000248 (^{0}.rpm.(min/mm)^2), 3(^{0}), 1400 (rpm.110 (mm/min).</td></tr>
<tr><td>Formula 36
<table>
<tr><td>Level</td><td>A/A^2</td><td>A/B^2</td><td>A/C^2</td><td>A^2</td><td>B^2</td><td>C^2</td></tr>
<tr><td>1</td><td>0.5</td><td>4.08E-06</td><td>0.00125</td><td>4</td><td>490000</td><td>1600</td></tr>
<tr><td>2</td><td>0.4</td><td>2.5E-06</td><td>0.000391</td><td>6.25</td><td>1000000</td><td>6400</td></tr>
<tr><td>3</td><td>0.3333</td><td>1.53E-06</td><td>0.0002</td><td>9</td><td>1960000</td><td>12100</td></tr>
</table>
Optimal parametric setting: $A/A^2 3 A/B^2 3 A/C^2 3 A^2 3 B^2 3 C^2 3$</td>
<td>joint (1/TTA^2), (TTA/TRS^2), (TTA/WS^2), (TTA^2) (TRS^2) and (WS^2) yield (1/TTA^2)3, (TTA/TRS^2)3, (TTA/WS^2)3 (TTA^2)3 (TRS^2)3 (WS^2)3 to obtain 3(1/^{0}), 1.5306E-06 (^{0}/rpm), 0.000248 (^{0}.rpm.(min/mm)^2), 9(^{0}), 0(rpm.12100 (mm/min)^2.</td></tr>
<tr><td>Formula 37
<table>
<tr><td>Level</td><td>B/A^2</td><td>B/B^2</td><td>B/C^2</td></tr>
<tr><td>1</td><td>175</td><td>0.0014</td><td>0.4375</td></tr>
<tr><td>2</td><td>160</td><td>0.0010</td><td>0.1563</td></tr>
<tr><td>3</td><td>155.5556</td><td>0.0007</td><td>0.1157</td></tr>
</table>
Optimal parametric setting: $B/A^2 1 B/B^2 2 B/C^2 1$</td>
<td>joint (TRS/TTA^2), (TRS/TRS^2), and (TRS/WS^2) yield (TRS/TTA^2)1, (TRS/TRS^2)2, (TRS/WS^2)1 to obtain 175 (rpm/^{0}), 0.001 (1/rpm), 0.4375 (rpm.(min/mm)^2.</td></tr>
<tr><td>Formula 38
<table>
<tr><td>Level</td><td>B/A^2</td><td>B/B^2</td><td>B/C^2</td><td>A</td><td>B</td></tr>
<tr><td>1</td><td>175</td><td>0.001429</td><td>0.4375</td><td>2</td><td>700</td></tr>
<tr><td>2</td><td>160</td><td>0.001</td><td>0.15625</td><td>2.5</td><td>1000</td></tr>
<tr><td>3</td><td>155.5556</td><td>0.000714</td><td>0.115702</td><td>3</td><td>1400</td></tr>
</table>
Optimal parametric setting: $B/A^2 1$ $B/B^2 1$ $B/C^2 1$ A3 B3 C3; $B/A^2 1$ $B/B^2 3$ $B/C^2 1$ A3 B3 C3</td>
<td>joint (TRS/TTA^2), (TRS/TRS^2), (TRS/WS^2), (TTA) (TRS) and (WS) yield (TRS/TTA^2)1, (TRS/TRS^2)1, (TRS/WS^2)1, (TTA)3 (TRS)3 (WS)3 to obtain 175 (rpm/^{0}), 0.001429 (1/rpm), 0.4375 (rpm.(min/mm)^2), 3(^{0}), 1400 (rpm). 110 (mm/min).
joint (TRS/TTA^2), (TRS/TRS^2), (TRS/WS^2), (TTA) (TRS) and (WS) yield (TRS/TTA^2)1, (TRS/TRS^2)3, (TRS/WS^2)1, (TTA)3 (TRS)3 (WS)3 to obtain 175 (rpm/^{0}), 155.5556 (1/rpm), 0.4375 (rpm.(min/mm)^2), 3(^{0}), 1400 (rpm). 110 (mm/min).</td></tr>
<tr><td>Formula 39
<table>
<tr><td>Level</td><td>B/A^2</td><td>B/B^2</td><td>B/C^2</td><td>A^2</td><td>B^2</td><td>C^2</td></tr>
<tr><td>1</td><td>175</td><td>0.0143</td><td>0.4375</td><td>4</td><td>490000</td><td>1600</td></tr>
<tr><td>2</td><td>160</td><td>0.001</td><td>0.1563</td><td>6.25</td><td>1000000</td><td>6400</td></tr>
<tr><td>3</td><td>155.5556</td><td>0.0007</td><td>0.1157</td><td>9</td><td>1960000</td><td>12100</td></tr>
</table>
Optimal parametric setting: $B/A^2 3$ $B/B^2 1$ $B/C^2 1$ $A^2 1$ $B^2 3$ $C^2 3$; $B/A^2 3$ $B/B^2 2$ $B/C^2 2$ $A^2 2$ $B^2 3$ $C^2 3$; $B/A^2 3$ $B/B^2 3$ $B/C^2 3$ $A^2 3$ $B^2 3$ $C^2 3$</td>
<td>joint (TRS/TTA^2), (TRS/TRS^2), (TRS/WS^2), (TTA^2) (TRS^2) and (WS^2) yield (TRS/TTA^2)3, (TRS/TRS^2)1, (TRS/WS^2)1, (TTA^2)1 (TRS^2)3 (WS^2)3 to obtain 175 (rpm/^{0}), 0.0143 (1/rpm), 0.4375 (rpm.(min/mm)^2), 4(^{0}), 1960000 (rpm). 12100 (mm/min)^2.
joint (TRS/TTA^2), (TRS/TRS^2), (TRS/WS^2), (TTA^2) (TRS^2) and (WS^2) yield (TRS/TTA^2)3, (TRS/TRS^2)2, (TRS/WS^2)2, (TTA^2)1 (TRS^2)2, (WS^2)3 to obtain 175 (rpm/^{0}), 0.001 (1/rpm), 0.15625 (rpm.(min/mm)^2), 6.25 (^{0}), 1960000 (rpm). 12100 (mm/min)^2.
joint (TRS/TTA^2), (TRS/TRS^2), (TRS/WS^2), (TTA^2) (TRS^2) and (WS^2) yield (TRS/TTA^2)3, (TRS/TRS^2)3, (TRS/WS^2)3, (TTA^2)3 (TRS^2)3, (WS^2)3 to obtain 175 (rpm/^{0}), 0.000714 (1/rpm), 0.115702 (rpm.(min/mm)^2), 9 (^{0}), 1960000 (rpm). 12100 (mm/min)^2.</td></tr>
</table>

<table>
<tr><th>Formulation</th><th>Optimal parametric setting and interpretation</th></tr>
<tr><td>Formula 40
<table>
<tr><td>Level</td><td>C/A^2</td><td>C/B^2</td><td>C/C^2</td></tr>
<tr><td>1</td><td>10</td><td>8.16327E-05</td><td>0.025</td></tr>
<tr><td>2</td><td>12.8</td><td>0.00008</td><td>0.0125</td></tr>
<tr><td>3</td><td>12.2222</td><td>5.6122E-05</td><td>0.0091</td></tr>
</table>
Optimal parametric setting: $C/A^2 2$ $C/B^2 1$ $C/C^2 1$</td>
<td>joint (WS/TTA^2), (WS/TRS^2), and (WS/WS^2) yield (WS/TTA^2)2, (WS/TRS^2)1, (WS/WS^2)1 to obtain 12.8 (^{0}mm/min), 8.16327E-05 (rpm.mm/min), 0.025 (min/mm).</td></tr>
<tr><td>Formula 41
<table>
<tr><td>Level</td><td>C/A^2</td><td>C/B^2</td><td>C/C^2</td><td>A</td><td>B</td><td>C</td></tr>
<tr><td>1</td><td>10</td><td>8.16E-05</td><td>0.025</td><td>2</td><td>700</td><td>40</td></tr>
<tr><td>2</td><td>12.8</td><td>0.00008</td><td>0.0125</td><td>2.5</td><td>1000</td><td>80</td></tr>
<tr><td>3</td><td>12.2222</td><td>5.61E-05</td><td>0.0091</td><td>3</td><td>1400</td><td>110</td></tr>
</table>
Optimal parametric setting: $C/A^2 3C/B^2 3C/C^2 3A3B3C3$</td>
<td>joint (WS/TTA^2), (WS/TRS^2), (WS/WS^2), (TTA) (TRS) and (WS) yield (WS/TTA^2)3, (WS/TRS^2)3, (WS/WS^2)3, (TTA)3 (TRS)3, (WS)3 to obtain 12.22222 (^{0}mm/min), 5.61E-05 (rpm.mm/min), 0.009091 (min/mm), 3(^{0}), 1400 (rpm), 110 (mm/min).</td></tr>
<tr><td>Formula 42
<table>
<tr><td>Level</td><td>C/A^2</td><td>C/B^2</td><td>C/C^2</td><td>A^2</td><td>B^2</td><td>C^2</td></tr>
<tr><td>1</td><td>10</td><td>8.16E-05</td><td>0.025</td><td>4</td><td>490000</td><td>1600</td></tr>
<tr><td>2</td><td>12.8</td><td>0.00008</td><td>0.0125</td><td>6.25</td><td>1000000</td><td>6400</td></tr>
<tr><td>3</td><td>12.22222</td><td>5.61E-05</td><td>0.009091</td><td>9</td><td>1960000</td><td>12100</td></tr>
</table>
Optimal parametric setting: $C/A^2 3C/B^2 3C/C^2 3A^2 3B^2 3C^2 3$</td>
<td>joint (WS/TTA^2), (WS/TRS^2), (WS/WS^2), (TTA^2) (TRS^2) and (WS^2) yield (WS/TTA^2)3, (WS/TRS^2)3, (WS/WS^2)3, (TTA^2)3 (TRS^2)3 (WS^2)3 to obtain 12.22222 (^{0}mm/min), 5.61E-05 (rpm.mm/min), 0.009091 (min/mm), 9(^{0}), 1960000 (rpm), 12100 (mm/min)^2.</td></tr>
<tr><td>Formula 43
<table>
<tr><td>Level</td><td>BC</td><td>AC</td><td>AB</td></tr>
<tr><td>1</td><td>28000</td><td>80</td><td>1400</td></tr>
<tr><td>2</td><td>80000</td><td>200</td><td>2500</td></tr>
<tr><td>3</td><td>154000</td><td>330</td><td>4200</td></tr>
</table>
Optimal parametric setting: BC3AC3AB3</td>
<td>joint (TRS.WS), (TTA.WS), and (TTA.TRS) yield (TRS.WS)3, (TTA.WS)3, (TTA.TRS)3 to obtain 154000 (rpm.mm/min), 330(^{0}.mm/min), 4200 (^{0}.rpm).</td></tr>
<tr><td>Formula 44
<table>
<tr><td>Level</td><td>BC</td><td>AC</td><td>AB</td><td>A</td><td>B</td><td>C</td></tr>
<tr><td>1</td><td>28000</td><td>80</td><td>1400</td><td>2</td><td>700</td><td>40</td></tr>
<tr><td>2</td><td>80000</td><td>200</td><td>2500</td><td>2.5</td><td>1000</td><td>80</td></tr>
<tr><td>3</td><td>154000</td><td>330</td><td>4200</td><td>3</td><td>1400</td><td>110</td></tr>
</table>
Optimal parametric setting: BC3AC3AB3A3B3C3</td>
<td>joint (TRS.WS), (TTA.WS), (TTA.TRS), (TTA), (TRS) and (WS) yield (TRS.WS)3, (TTA.WS)3, (TTA.TRS)3, (TTA)3 (TRS)3, (WS)3 to obtain 154000 (rpm.mm/min), 330(^{0}.mm/min), 4200 (^{0}.rpm), 3(^{0}), 1400 (rpm), 110 (mm/min).</td></tr>
<tr><td>Formula 45
<table>
<tr><td>Level</td><td>BC</td><td>AC</td><td>AB</td><td>A^2</td><td>B^2</td><td>C^2</td></tr>
<tr><td>1</td><td>28000</td><td>80</td><td>1400</td><td>4</td><td>490000</td><td>1600</td></tr>
<tr><td>2</td><td>80000</td><td>200</td><td>2500</td><td>6.25</td><td>1000000</td><td>6400</td></tr>
<tr><td>3</td><td>154000</td><td>330</td><td>4200</td><td>9</td><td>1960000</td><td>12100</td></tr>
</table>
Optimal parametric setting: $BC3AC3AB2A^2 3B^2 3C^2 3$</td>
<td>joint (TRS.WS), (TTA.WS), (TTA.TRS), (TTA^2), (TRS^2) and (WS^2) yield (TRS.WS)3, (TTA.WS)3, (TTA.TRS)3, (TTA^2)3 (TRS^2)3, (WS^2)3 to obtain 154000 (rpm.mm/min), 330(^{0}.mm/min), 4200 (^{0}.rpm), 9(^{0}), 1960000 (rpm), 12100 (mm/min)^2.</td></tr>
<tr><td>Formula 46
<table>
<tr><td>Level</td><td>AB^2</td><td>A^2B</td><td>$(AB)^2/C$</td></tr>
<tr><td>1</td><td>980000</td><td>2800</td><td>49000</td></tr>
<tr><td>2</td><td>2500000</td><td>6250</td><td>78125</td></tr>
<tr><td>3</td><td>5880000</td><td>12600</td><td>160363.6</td></tr>
</table>
Optimal parametric setting: $AB^2 3A^2B3(AB)^2/C3$</td>
<td>joint (TTA.WS^2), (TTA^2.TRS), and (TTA.TRS)^2/WS yield (TTA.WS^2)3, (TTA^2.TRS)3, ((TTA.TRS)^2/WS)3 to obtain 154000 (^{0}.rpm), 330(^{0}.rpm), 4200 (^{0}.rpm.min/mm).</td></tr>
<tr><td>Formula 47
<table>
<tr><td>Level</td><td>AB^2</td><td>A^2B</td><td>$(AB)^2/C$</td><td>A</td><td>B</td><td>C</td></tr>
<tr><td>1</td><td>980000</td><td>2800</td><td>49000</td><td>2</td><td>700</td><td>40</td></tr>
<tr><td>2</td><td>2500000</td><td>6250</td><td>78125</td><td>2.5</td><td>1000</td><td>80</td></tr>
<tr><td>3</td><td>5880000</td><td>12600</td><td>160363.6</td><td>3</td><td>1400</td><td>110</td></tr>
</table>
Optimal parametric setting: $AB^2 3A^2B3(AB)^2/C3A3B3C3$</td>
<td>joint (TTA.WS^2), (TTA^2.TRS), (TTA.TRS)^2/WS, (TTA), (TRS) and (WS) yield (TTA.WS^2)3, (TTA^2.TRS)3, ((TTA.TRS)^2/WS)3, (TTA)3 (TRS)3, (WS)3 to obtain 154000 (^{0}.rpm), 12600 (^{0}.rpm), 160363.6 (^{0}.rpm.min/mm), 3(^{0}), 1400 (rpm), 110 (mm/min).</td></tr>
</table>

<table>
<tr><th>Formulation</th><th>Optimal parametric setting and interpretation</th></tr>
<tr><td>Formula 48
<table>
<tr><td>Level</td><td>AB^2</td><td>A^2B</td><td>$(AB)^2/C$</td><td>A^2</td><td>B^2</td><td>C^2</td></tr>
<tr><td>1</td><td>980000</td><td>2800</td><td>49000</td><td>4</td><td>490000</td><td>1600</td></tr>
<tr><td>2</td><td>2500000</td><td>6250</td><td>78125</td><td>6.25</td><td>1000000</td><td>6400</td></tr>
<tr><td>3</td><td>5880000</td><td>12600</td><td>160363.6</td><td>9</td><td>1960000</td><td>12100</td></tr>
</table>
Optimal parametric setting: $AB^23A^2B3(AB)^2/C3A^23B^23C^23$</td>
<td>joint (TTA.WS^2), (TTA^2.TRS), (TTA.TRS)^2/WS, (TTA^2), (TRS^2) and (WS^2) yield (TTA.WS^2)3, (TTA^2.TRS)3, ((TTA.TRS)^2/WS)3, (TTA^2)3 (TRS^2)3, (WS^2)3 to obtain 154000 (^{0}.rpm), 12600 (^{0}.rpm), 160363.6 (^{0}.rpm.min/mm), 9(^{0}), 1960000 (rpm), 12100 (mm/min)^2.</td></tr>
<tr><td>Formula 49
<table>
<tr><td>Level</td><td>AC^2</td><td>$(AC)^2/B$</td><td>(A^2C)</td></tr>
<tr><td>1</td><td>3200</td><td>9.1429</td><td>160</td></tr>
<tr><td>2</td><td>16000</td><td>40</td><td>500</td></tr>
<tr><td>3</td><td>36300</td><td>77.7857</td><td>990</td></tr>
</table>
Optimal parametric setting: $AC^23AC^2/B3A^2C3$</td>
<td>Joint (TTA.WS^2), ((TTA.WS)^2/TRS), and (TTA)^2.WS yield (TTA.WS^2)3, ((TTA.WS)^2/TRS)3, (TTA)^2.WS)3 to obtain 36300 (^{0}.(mm/min)^2), 77.78571 (^{0}.mm/rpm.min), 990 (^{0}.mm/min).</td></tr>
<tr><td>Formula 50
<table>
<tr><td>Level</td><td>AC^2</td><td>$(AC)^2/B$</td><td>(A^2C)</td><td>A</td><td>B</td><td>C</td></tr>
<tr><td>1</td><td>3200</td><td>9.142857</td><td>160</td><td>2</td><td>700</td><td>40</td></tr>
<tr><td>2</td><td>16000</td><td>40</td><td>500</td><td>2.5</td><td>1000</td><td>80</td></tr>
<tr><td>3</td><td>36300</td><td>77.78571</td><td>990</td><td>3</td><td>1400</td><td>110</td></tr>
</table>
Optimal parametric setting: $AC^23AC^2/B3A^2C3A3B3C3$</td>
<td>Joint (TTA.WS^2), ((TTA.WS)^2/TRS), (TTA)^2.WS, (TTA), (TRS) and (WS) yield (TTA.WS^2)3, ((TTA.WS)^2/TRS)3, (TTA)^2.WS)3, (TTA)3 (TRS)3, (WS)3 to obtain 36300 (^{0}.(mm/min)^2), 77.78571 (^{0}.mm/rpm.min), 990 (^{0}.mm/min), 3(^{0}), 1400 (rpm), 110 (mm/min).</td></tr>
<tr><td>Formula 51
<table>
<tr><td>Level</td><td>AC^2</td><td>$(AC)^2/B$</td><td>(A^2C)</td><td>A^2</td><td>B^2</td><td>C^2</td></tr>
<tr><td>1</td><td>3200</td><td>9.1429</td><td>160</td><td>4</td><td>490000</td><td>1600</td></tr>
<tr><td>2</td><td>16000</td><td>40</td><td>500</td><td>6.25</td><td>1000000</td><td>6400</td></tr>
<tr><td>3</td><td>36300</td><td>77.7857</td><td>990</td><td>9</td><td>1960000</td><td>12100</td></tr>
</table>
Optimal parametric setting: $AC^23AC^2/B3A^2C3A^23B^23C^23$</td>
<td>Joint (TTA.WS^2), ((TTA.WS)^2/TRS), (TTA)^2.WS, (TTA^2), (TRS^2) and (WS^2) yield (TTA.WS^2)3, ((TTA.WS)^2/TRS)3, (TTA)^2.WS)3, (TTA^2)3 (TRS^2)3, (WS^2)3 to obtain 36300 (^{0}.(mm/min)^2), 77.78571 (^{0}.mm/rpm.min), 990 (^{0}.mm/min), 9(^{0}), 1960000 (rpm), 12100 (mm/min)^2.</td></tr>
<tr><td>Formula 52
<table>
<tr><td>Level</td><td>BC/A</td><td>C</td><td>B</td></tr>
<tr><td>1</td><td>14000</td><td>40</td><td>700</td></tr>
<tr><td>2</td><td>32000</td><td>80</td><td>1000</td></tr>
<tr><td>3</td><td>51333.33</td><td>110</td><td>1400</td></tr>
</table>
Optimal parametric setting: BC/A3C3B3</td>
<td>Joint TRS.WS/TTA, WS, and TRS yield (TRS.WS/TTA)3, (WS)3, (TRS)3 to obtain 51333.33 (rpm.mm/min^0), 110 (mm/min), 1400 (rpm).</td></tr>
<tr><td>Formula 53
<table>
<tr><td>Level</td><td>BC/A</td><td>C</td><td>B</td><td>A</td><td>B</td><td>C</td></tr>
<tr><td>1</td><td>14000</td><td>40</td><td>700</td><td>2</td><td>700</td><td>40</td></tr>
<tr><td>2</td><td>32000</td><td>80</td><td>1000</td><td>2.5</td><td>1000</td><td>80</td></tr>
<tr><td>3</td><td>51333.33</td><td>110</td><td>1400</td><td>3</td><td>1400</td><td>110</td></tr>
</table>
Optimal parametric setting: BC/A3C3B3A3B3C3</td>
<td>Joint TRS.WS/TTA, WS, TRS, (TTA), (TRS) and (WS) yield (TRS.WS/TTA)3, (WS)3, (TRS)3, (TTA)3 (TRS)3, (WS)3 to obtain 51333.33 (rpm.mm/min.^{0}), 110 (mm/min), 1400 (rpm), 3(^{0}), 1400 (rpm), 110 (mm/min).</td></tr>
<tr><td>Formula 54
<table>
<tr><td>Level</td><td>BC/A</td><td>C</td><td>B</td><td>A^2</td><td>B^2</td><td>C^2</td></tr>
<tr><td>1</td><td>14000</td><td>40</td><td>700</td><td>4</td><td>490000</td><td>1600</td></tr>
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<tr><td>3</td><td>51333.33</td><td>110</td><td>1400</td><td>9</td><td>1960000</td><td>12100</td></tr>
</table>
Optimal parametric setting: $BC/A3C3B3A^23B^23C^23$</td>
<td>Joint TRS.WS/TTA, WS, TRS, (TTA^2), (TRS^2) and (WS^2) yield (TRS.WS/TTA)3, (WS)3, (TRS)3, (TTA^2)3, (TRS^2)3, (WS^2)3 to obtain 51333.33 (rpm.mm/min.^{0}), 110 (mm/min), 1400 (rpm), 9(^{0}), 1960000 (rpm), 12100 (mm/min)^2.</td></tr>
<tr><td>Formula 55
<table>
<tr><td>Level</td><td>$(BC)^2/A$</td><td>BC^2</td><td>B^2C</td></tr>
<tr><td>1</td><td>3.92E+08</td><td>1120000</td><td>19600000</td></tr>
<tr><td>2</td><td>2.56E+09</td><td>6400000</td><td>80000000</td></tr>
<tr><td>3</td><td>7.91E+09</td><td>1.7E+07</td><td>2.16E+08</td></tr>
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Optimal parametric setting: $(BC)^2/A3BC^23B^2C3$</td>
<td>Joint (TRS.WS)^2/TTA, TRS.(WS)^2, and (TRS)^2.WS Yield (TRS.WS)^2/TTA, TRS.(WS)^2, and (TRS)^2.WS to obtain 7.91E+09 (rpm.(mm)^2/(min)^2. ^{0}), 1.7E+07 (mm/min)^2.rpm, 2.16E+08 (rpm.mm/min).</td></tr>
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<table>
<tr><th>Formulation</th><th>Optimal parametric setting and interpretation</th></tr>
<tr><td>Formula 56
<table>
<tr><td>Level</td><td>$(BC)^2/A$</td><td>BC^2</td><td>B^2C</td><td>A</td><td>B</td><td>C</td></tr>
<tr><td>1</td><td>3.92E+08</td><td>1120000</td><td>19600000</td><td>2</td><td>700</td><td>40</td></tr>
<tr><td>2</td><td>2.56E+09</td><td>6400000</td><td>80000000</td><td>2.5</td><td>1000</td><td>80</td></tr>
<tr><td>3</td><td>7.91E+09</td><td>16940000</td><td>2.16E+08</td><td>3</td><td>1400</td><td>110</td></tr>
</table>
Optimal parametric setting: $(BC)^2/A3BC^23B^2C3A3B3C3$</td><td>Joint (TRS.WS)^2/TTA, TRS.(WS)^2, (TRS)^2.WS, (TTA), (TRS) and (WS) Yield ((TRS.WS)^2/TTA)3, (TRS.(WS)^2)3, ((TRS)^2.WS)3, (TTA)3, (TRS)3, (WS)3 to obtain 7.91E+09 (rpm.(mm)^2/(min)^2. ^{0}), 16940000 (mm/min)^2.rpm, 2.16E+08 (rpm.mm/min), 3(^{0}), 1400 (rpm), 110 (mm/min).</td></tr>
<tr><td>Formula 57
<table>
<tr><td>Level</td><td>$(BC)^2/A$</td><td>BC^2</td><td>B^2C</td><td>A^2</td><td>B^2</td><td>C^2</td></tr>
<tr><td>1</td><td>3.92E+08</td><td>1120000</td><td>19600000</td><td>4</td><td>490000</td><td>1600</td></tr>
<tr><td>2</td><td>2.56E+09</td><td>6400000</td><td>80000000</td><td>6.25</td><td>1000000</td><td>6400</td></tr>
<tr><td>3</td><td>7.91E+09</td><td>16940000</td><td>2.16E+08</td><td>9</td><td>1960000</td><td>12100</td></tr>
</table>
Optimal parametric setting: $(BC)^2/A3$ BC^23 B^2C3 A^23 B^23 C^23</td><td>Joint (TRS.WS)^2/TTA, TRS.(WS)^2, (TRS)^2.WS, (TTA^2), (TRS^2) and (WS^2) Yield ((TRS.WS)^2/TTA)3, (TRS.(WS)^2)3, ((TRS)^2.WS)3, (TTA^2)3, (TRS^2)3 (WS^2)3 to obtain 7.91E+09 (rpm.(mm)^2/(min)^2. ^{0}), 16940000 (mm/min)^2.rpm, 2.16E+08 (rpm.mm/min), 9(^{0}), 1960000 (rpm), 12100 (mm/min)^2.</td></tr>
<tr><td>Formula 58
<table>
<tr><td>Level</td><td>AB</td><td>A^2</td><td>A^2B/C</td></tr>
<tr><td>1</td><td>1400</td><td>4</td><td>70</td></tr>
<tr><td>2</td><td>2500</td><td>6.25</td><td>78.125</td></tr>
<tr><td>3</td><td>4200</td><td>9</td><td>114.5455</td></tr>
</table>
Optimal parametric setting: AB3 A^23 $(A^2B/C)3$</td><td>Joint (TTA.TRS), (TTA)^2, and (TTA^2TRS)/WS Yield (TTA.TRS)3, ((TTA)^2)3, ((TTA^2TRS)/WS)3 to obtain 4200 (^{0}.rpm), 9 (^{0}), 114.5455 (^{0}.rpm.min/mm).</td></tr>
<tr><td>Formula 59
<table>
<tr><td>Level</td><td>AB</td><td>A^2</td><td>A^2B/C</td><td>A</td><td>B</td><td>C</td></tr>
<tr><td>1</td><td>1400</td><td>4</td><td>70</td><td>2</td><td>700</td><td>40</td></tr>
<tr><td>2</td><td>2500</td><td>6.25</td><td>78.125</td><td>2.5</td><td>1000</td><td>80</td></tr>
<tr><td>3</td><td>4200</td><td>9</td><td>114.5455</td><td>3</td><td>1400</td><td>110</td></tr>
</table>
Optimal parametric setting: AB3 A^23 $A^2B/C3$ A3 B3 C3</td><td>Joint (TTA.TRS), (TTA)^2, (TTA^2TRS)/WS, (TTA), (TRS) and (WS) Yield (TTA.TRS)3, ((TTA)^2)3, ((TTA^2TRS)/WS)3, (TTA)3, (TRS)3, (WS)3 to obtain 4200 (^{0}.rpm), 9 (^{0}), 114.5455 (^{0}.rpm.min/mm), 3(^{0}), 1400 (rpm), 110 (mm/min).</td></tr>
<tr><td>Formula 60
<table>
<tr><td>Level</td><td>AB</td><td>A^2</td><td>A^2B/C</td><td>A^2</td><td>B^2</td><td>C^2</td></tr>
<tr><td>1</td><td>1400</td><td>4</td><td>70</td><td>4</td><td>490000</td><td>1600</td></tr>
<tr><td>2</td><td>2500</td><td>6.25</td><td>78.125</td><td>6.25</td><td>1000000</td><td>6400</td></tr>
<tr><td>3</td><td>4200</td><td>9</td><td>114.5455</td><td>9</td><td>1960000</td><td>12100</td></tr>
</table>
Optimal parametric setting: AB3 A^23 $A^2B/C3$ A^23 B^23 C^23</td><td>Joint (TTA.TRS), (TTA)^2, (TTA^2TRS)/WS, (TTA^2), (TRS^2) and (WS^2) Yield (TTA), (TTA.TRS)3, ((TTA)^2)3, ((TTA^2TRS)/WS)3, (TTA^2)3, (TRS^2)3 (WS^2)3 to obtain 4200 (^{0}.rpm), 9 (^{0}), 114.5455 (^{0}.rpm.min/mm), 9(^{0}), 1960000 (rpm), 12100 (mm/min)^2.</td></tr>
<tr><td>Formula 61
<table>
<tr><td>Level</td><td>AC</td><td>A^2C/B</td><td>A^2</td></tr>
<tr><td>1</td><td>80</td><td>0.2286</td><td>4</td></tr>
<tr><td>2</td><td>200</td><td>0.5</td><td>6.25</td></tr>
<tr><td>3</td><td>303</td><td>0.7071</td><td>9</td></tr>
</table>
Optimal parametric setting: AC3 A^2C/B A^2</td><td>Joint TTA.WS/TTA, (TTA)^2.WS/TRS and (TTA)^2 yield TTA.WS/TTA, (TTA)^2.WS/TRS and (TTA)^2 to obtain 303(^{0}.mm/min), 0.707143 (^{0}.mm/min.rpm), 9 (^{0}).</td></tr>
<tr><td>Formula 62
<table>
<tr><td>Level</td><td>AC</td><td>A^2C/B</td><td>A^2</td><td>A</td><td>B</td><td>C</td></tr>
<tr><td>1</td><td>80</td><td>0.2286</td><td>4</td><td>2</td><td>700</td><td>40</td></tr>
<tr><td>2</td><td>200</td><td>0.5</td><td>6.25</td><td>2.5</td><td>1000</td><td>80</td></tr>
<tr><td>3</td><td>330</td><td>0.7071</td><td>9</td><td>3</td><td>1400</td><td>110</td></tr>
</table>
Optimal parametric setting: AB3 A^23 $A^2B/C3$ A3 B3 C3</td><td>Joint TTA.WS/TTA, (TTA)^2.WS/TRS, (TTA)^2, (TTA) (TRS) and (WS)
Yield (TTA.WS/TTA)3, ((TTA)^2.WS/TRS)3, ((TTA)^2)3, (TTA)3 (TRS)3 (WS)3 to obtain 303(^{0}.mm/min), 0.707143 (^{0}.mm/min.rpm), 9 (^{0}), 9 (^{0}), 1960000 (rpm), 12100 (mm/min).</td></tr>
</table>

<table>
<tr><th>Formulation</th><th>Optimal parametric setting and interpretation</th></tr>
<tr><td>Formula 63
<table>
<tr><td>Level</td><td>AC</td><td>A^2C/B</td><td>A</td><td>A^2</td><td>B^2</td><td>C^2</td></tr>
<tr><td>1</td><td>80</td><td>0.2286</td><td>4</td><td>4</td><td>490000</td><td>1600</td></tr>
<tr><td>2</td><td>200</td><td>0.5</td><td>6.25</td><td>6.25</td><td>1000000</td><td>6400</td></tr>
<tr><td>3</td><td>330</td><td>0.7071</td><td>9</td><td>9</td><td>1960000</td><td>12100</td></tr>
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Optimal parametric setting: AB3 $A^2$3 A^2B/C3 $A^2$3 $B^2$3 $C^2$3</td>
<td>Joint TTA.WS/TTA, (TTA)^2.WS/TRS, (TTA)^2, (TTA)^2, (TRS)^2 and (WS)^2 Yield (TTA.WS/TTA)3, ((TTA)^2).WS/TRS)3, ((TTA)^2)3, (TTA^2)3 (TRS^2)3 (WS^2)3 to obtain 303(^{0}.mm/min), 0.707143 (^{0}.mm/min.rpm), 9 (^{0}), 9(^{0}), 1960000 (rpm), 12100 (mm/min)^2.</td></tr>
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<table>
<tr><td>Level</td><td>B^2C/A</td><td>BC</td><td>B^2</td></tr>
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<tr><td>2</td><td>32000000</td><td>80000</td><td>1000000</td></tr>
<tr><td>3</td><td>71866667</td><td>154000</td><td>1960000</td></tr>
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Optimal parametric setting: B^2C/A3 BC3 $B^2$3</td>
<td>Joint (TRS)^2.WS/TTA, (TRS.WS) and (TRS)^2 yield ((TRS)^2.WS/TTA)3, (TRS.WS)3, ((TRS)^2)3 to obtain 71866667 (rpm)^2mm/min.^{0}), 154000 (rpm.^{0}), 1960000 (rpm.^{0}).</td></tr>
<tr><td>Formula 65
<table>
<tr><td>Level</td><td>B^2C/A</td><td>BC</td><td>B^2</td><td>A</td><td>B</td><td>C</td></tr>
<tr><td>1</td><td>9800000</td><td>28000</td><td>490000</td><td>2</td><td>700</td><td>40</td></tr>
<tr><td>2</td><td>32000000</td><td>80000</td><td>1000000</td><td>2.5</td><td>1000</td><td>80</td></tr>
<tr><td>3</td><td>71866667</td><td>154000</td><td>1960000</td><td>3</td><td>1400</td><td>110</td></tr>
</table>
Optimal parametric setting: B^2C/A3 BC3 $B^2$3 A3 B1 C1; B2C/A3 BC3 B23 A3 B2 C2; B2C/A3 BC3 B23 A3 B3 C3</td>
<td>Joint (TRS)^2.WS/TTA, (TRS.WS), (TRS)^2, (TTA) (TRS) and (WS).
Yield ((TRS)^2.WS/TTA)3, (TRS.WS)3, ((TRS)^2)3, (TTA)3, (TRS)1, (WS)1 to obtain 71866667 (rpm)^2mm/min.^{0}), 154000 (rpm.^{0}), 1960000 (rpm.^{0}), 3(^{0}), 700 (rpm), 40 (mm/min).
Joint (TRS)^2.WS/TTA, (TRS.WS), (TRS)^2, (TTA) (TRS) and (WS).
Yield ((TRS)^2.WS/TTA)3, (TRS.WS)3, ((TRS)^2)3, (TTA)3, (TRS)3, (WS)3 to obtain 71866667 (rpm)^2mm/min.^{0}), 154000 (rpm.^{0}), 1960000 (rpm.^{0}), 3(^{0}), 1400 (rpm), 110 (mm/min).</td></tr>
<tr><td>Formula 66
<table>
<tr><td>Level</td><td>B^2C/A</td><td>BC</td><td>B^2</td><td>A^2</td><td>B^2</td><td>C^2</td></tr>
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Optimal parametric setting: B^2C/A3 BC3 $B^2$3 $A^2$3 $B^2$3 $C^2$3</td>
<td>Joint (TRS)^2.WS/TTA, (TRS.WS), (TRS)^2, (TTA)^2, (TRS)^2 and (WS)^2.
Yield ((TRS)^2.WS/TTA)3, (TRS.WS)3, ((TRS)^2)3, ((TTA)^2)3, ((TRS)^2)3, ((WS)^2)3 to obtain 71866667 (rpm)^2mm/min.^{0}), 154000 (rpm.^{0}), 1960000 (rpm.^{0}), 9(^{0}), 1960000 (rpm), 12100 (mm/min)^2.</td></tr>
<tr><td>Formula 67
<table>
<tr><td>Level</td><td>C^2</td><td>C^2A/B</td><td>CA</td></tr>
<tr><td>1</td><td>1600</td><td>4.5714</td><td>80</td></tr>
<tr><td>2</td><td>6400</td><td>16</td><td>200</td></tr>
<tr><td>3</td><td>12100</td><td>25.9286</td><td>330</td></tr>
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Optimal parametric setting: $C^2$3 C^2A/B3 CA3</td>
<td>Joint (WS)^2, ((TTA.(WS)^2/TRS) and (WS.TTA) yield ((WS)^2)3, ((TTA.(WS)^2/TRS)3, (WS.TTA)3 to obtain 71866667 (mm/min)^2, 154000 (mm/min)^2.^{0}), 1960000 (mm/min.^{0}).</td></tr>
<tr><td>Formula 68
<table>
<tr><td>Level</td><td>C^2</td><td>C^2A/B</td><td>CA</td><td>A</td><td>B</td><td>C</td></tr>
<tr><td>1</td><td>1600</td><td>4.5714</td><td>80</td><td>2</td><td>700</td><td>40</td></tr>
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<tr><td>3</td><td>12100</td><td>25.9286</td><td>330</td><td>3</td><td>1400</td><td>110</td></tr>
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Optimal parametric setting: $C^2$3 C^2A/B3 CA3 A3 B3 C3</td>
<td>Joint (WS)^2, ((TTA.(WS)^2/TRS) (WS.TTA), (TTA) (TRS) and (WS) yield ((WS)^2)3, ((TTA.(WS)^2/TRS)3, (WS.TTA)3, (TTA)3, (TRS)3, (WS)3 to obtain 12100 (mm/min)^2, 25.92857 (mm/min)^2.^{0}), 330 (mm/min.^{0}), 3(^{0}), 1400 (rpm), 110 (mm/min).</td></tr>
<tr><td>Formula 69
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<tr><td>Level</td><td>C^2</td><td>C^2A/B</td><td>CA</td><td>A^2</td><td>B^2</td><td>C^2</td></tr>
<tr><td>1</td><td>1600</td><td>4.5714</td><td>80</td><td>4</td><td>490000</td><td>1600</td></tr>
<tr><td>2</td><td>6400</td><td>16</td><td>200</td><td>6.25</td><td>1000000</td><td>6400</td></tr>
<tr><td>3</td><td>12100</td><td>25.9286</td><td>330</td><td>9</td><td>1960000</td><td>12100</td></tr>
</table>
Optimal parametric setting: $C^2$3 C^2A/B3 CA3 $A^2$3 $B^2$3 $C^2$3</td>
<td>Joint (WS)^2, ((TTA.(WS)^2/TRS) (WS.TTA), (TTA)^2, (TRS)^2 and (WS)^2 yield ((WS)^2)3, ((TTA.(WS)^2/TRS)3, (WS.TTA)3, ((TTA)^2)3, ((TRS)^2)3, ((WS)^2)3 to obtain 12100 (mm/min)^2, 25.92857 (mm/min)^2.^{0}), 330 (mm/min.^{0}), 9(^{0}), 1960000 (rpm), 12100 (mm/min)^2.</td></tr>
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<table>
<tr><th>Formulation</th><th>Optimal parametric setting and interpretation</th></tr>
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<tr><td>Level</td><td>ABC</td><td>A^2C</td><td>A^2B</td></tr>
<tr><td>1</td><td>56000</td><td>160</td><td>2800</td></tr>
<tr><td>2</td><td>200000</td><td>500</td><td>6250</td></tr>
<tr><td>3</td><td>462000</td><td>990</td><td>12600</td></tr>
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Optimal parametric setting: ABC3 A^2C3 A^2B3</td>
<td>Joint (TTA.TRS.WS), ((TTA.(WS)^2) and ((TTA)^2/WS) yield (TTA.TRS.WS)3, ((TTA.(WS)^2)3 and ((TTA)^2/WS)3 to obtain 462000 (^{0}.rpm.mm/min), 990 (^{0}.mm/min), 12600 (^{0}.rpm).</td></tr>
<tr><td>Formula 71
<table>
<tr><td>Level</td><td>ABC</td><td>A^2C</td><td>A^2B</td><td>A</td><td>B</td><td>C</td></tr>
<tr><td>1</td><td>56000</td><td>160</td><td>2800</td><td>2</td><td>700</td><td>40</td></tr>
<tr><td>2</td><td>200000</td><td>500</td><td>6250</td><td>2.5</td><td>1000</td><td>80</td></tr>
<tr><td>3</td><td>462000</td><td>990</td><td>12600</td><td>3</td><td>1400</td><td>110</td></tr>
</table>
Optimal parametric setting: ABC3 A^2C3 A^2B3 A3 B3 C3</td>
<td>Joint (TTA.TRS.WS), ((TTA.(WS)^2), ((TTA)^2/WS), (TTA) (TRS) and (WS) yield (TTA.TRS.WS)3, ((TTA.(WS)^2)3, ((TTA)^2/WS)3, (TTA)3 (TRS)3, (WS)3 to obtain 462000 (^{0}.rpm.mm/min), 990 (^{0}.mm/min), 12600 (^{0}.rpm), 3(^{0}), 1400 (rpm), 110 (mm/min).</td></tr>
<tr><td>Formula 72
<table>
<tr><td>Level</td><td>ABC</td><td>A^2C</td><td>A^2B</td><td>A^2</td><td>B^2</td><td>C^2</td></tr>
<tr><td>1</td><td>56000</td><td>160</td><td>2800</td><td>4</td><td>490000</td><td>1600</td></tr>
<tr><td>2</td><td>200000</td><td>500</td><td>6250</td><td>6.25</td><td>1000000</td><td>6400</td></tr>
<tr><td>3</td><td>462000</td><td>990</td><td>12600</td><td>9</td><td>1960000</td><td>12100</td></tr>
</table>
Optimal parametric setting: ABC1 A^2C3 A^2B3 A^23 B^23 C^23</td>
<td>Joint (TTA.TRS.WS), ((TTA.(WS)^2), ((TTA)^2/WS), (TTA)^2, (TRS)^2 and (WS)^2 yield (TTA.TRS.WS)3, ((TTA.(WS)^2)3, ((TTA)^2/WS)3, (TTA^2)3 (TRS^2)3 (WS^2)3 to obtain 462000 (^{0}.rpm.mm/min), 990 (^{0}.mm/min), 12600 (^{0}.rpm), 9(^{0}), 1960000 (rpm), 12100 (mm/min)^2.</td></tr>
<tr><td>Formula 73
<table>
<tr><td>Level</td><td>B^2C</td><td>BAC</td><td>B^2A</td></tr>
<tr><td>1</td><td>19600000</td><td>56000</td><td>980000</td></tr>
<tr><td>2</td><td>80000000</td><td>200000</td><td>2500000</td></tr>
<tr><td>3</td><td>2.16E+08</td><td>462000</td><td>5880000</td></tr>
</table>
Optimal parametric setting: B^2C3 BAC3 B^2A3</td>
<td>Joint ((TRS)^2.WS), (TTA.TRS.WS) and ((WS)^2.TTA) yield ((TRS)^2.WS)3, (TTA.TRS.WS)3, ((WS)^2.TTA)3 to obtain 2.16E+08 (rpm)^2.mm/min), 462000 (^{0}.rpm.mm/min), 5880000 (rpm)^2. ^{0}).</td></tr>
<tr><td>Formula 74
<table>
<tr><td>Level</td><td>B^2C</td><td>BAC</td><td>B^2A</td><td>A</td><td>B</td><td>C</td></tr>
<tr><td>1</td><td>19600000</td><td>56000</td><td>980000</td><td>2</td><td>700</td><td>40</td></tr>
<tr><td>2</td><td>80000000</td><td>200000</td><td>2500000</td><td>2.5</td><td>1000</td><td>80</td></tr>
<tr><td>3</td><td>2.16E+08</td><td>462000</td><td>5880000</td><td>3</td><td>1400</td><td>110</td></tr>
</table>
Optimal parametric setting: B^2C3 BAC3 B^2A3 A3 B3 C3</td>
<td>Joint ((TRS)^2.WS), (TTA.TRS.WS), ((WS)^2.TTA), (TTA) (TRS) and (WS) yield ((TRS)^2.WS)3, (TTA.TRS.WS)3, ((WS)^2.TTA)3, (TTA)3 (TRS)3, (WS)3 to obtain 2.16E+08 (rpm)^2.mm/min), 462000 (^{0}.rpm.mm/min), 5880000 (rpm)^2. ^{0}), 3(^{0}), 1400 (rpm), 110 (mm/min).</td></tr>
<tr><td>Formula 75
<table>
<tr><td>Level</td><td>B^2C</td><td>BAC</td><td>B^2A</td><td>A^2</td><td>B^2</td><td>C^2</td></tr>
<tr><td>1</td><td>19600000</td><td>56000</td><td>980000</td><td>4</td><td>490000</td><td>1600</td></tr>
<tr><td>2</td><td>80000000</td><td>200000</td><td>2500000</td><td>6.25</td><td>1000000</td><td>6400</td></tr>
<tr><td>3</td><td>2.16E+08</td><td>462000</td><td>5880000</td><td>9</td><td>1960000</td><td>12100</td></tr>
</table>
Optimal parametric setting: B^2C3 BAC3 B^2A3 A^23 B^23 C^23</td>
<td>Joint ((TRS)^2.WS), (TTA.TRS.WS), ((WS)^2.TTA), (TTA^2), (TRS^2), and (WS^2) yield ((TRS)^2.WS)3, (TTA.TRS.WS)3, ((WS)^2.TTA)3, (TTA^2)3 (TRS^2)3 (WS^2)3 to obtain 2.16E+08 (rpm)^2.mm/min), 462000 (^{0}.rpm.mm/min), 5880000 (rpm)^2. ^{0}), 9(^{0}), 1960000 (rpm), 12100 (mm/min).</td></tr>
<tr><td>Formula 76
<table>
<tr><td>Level</td><td>BC^2</td><td>AC^2</td><td>ABC</td></tr>
<tr><td>1</td><td>1120000</td><td>3200</td><td>56000</td></tr>
<tr><td>2</td><td>6400000</td><td>16000</td><td>200000</td></tr>
<tr><td>3</td><td>16940000</td><td>36300</td><td>462000</td></tr>
</table>
Optimal parametric setting: BC^23 AC^23 ABC3</td>
<td>Joint (TRS(WS)^2), ((TTA.(WS)^2) and (TTA.TRS.WS) yield (TRS(WS)^2)3 ((TTA.(WS)^2)3, (TTA.TRS.WS)3 to obtain 16940000 rpm(mm/min)^2, 36300 (mm/min)^2.^{0}), 462000 (rpm.mm/min.^{0}).</td></tr>
<tr><td>Formula 77
<table>
<tr><td>Level</td><td>BC^2</td><td>AC^2</td><td>ABC</td><td>A</td><td>B</td><td>C</td></tr>
<tr><td>1</td><td>1120000</td><td>3200</td><td>56000</td><td>2</td><td>700</td><td>40</td></tr>
<tr><td>2</td><td>6400000</td><td>16000</td><td>200000</td><td>2.5</td><td>1000</td><td>80</td></tr>
<tr><td>3</td><td>16940000</td><td>36300</td><td>462000</td><td>3</td><td>1400</td><td>110</td></tr>
</table>
Optimal parametric setting: BC^23 AC^23 ABC3 A3 B3 C3</td>
<td>Joint (TRS(WS)^2), ((TTA.(WS)^2), (TTA.TRS.WS), (TTA) (TRS) and (WS) yield (TRS(WS)^2)3 ((TTA.(WS)^2)3, (TTA.TRS.WS)3, (TTA)3 (TRS)3, (WS)3 to obtain 16940000 rpm(mm/min)^2, 36300 (mm/min)^2.^{0}), 462000 (rpm.mm/min.^{0}), 3(^{0}), 1400 (rpm), 110 (mm/min).</td></tr>
</table>

<table>
<tr><th>Formulation</th><th>Optimal parametric setting and interpretation</th></tr>
<tr><td>Formula 78
<table>
<tr><td>Level</td><td>BC^2</td><td>AC^2</td><td>ABC</td><td>A^2</td><td>B^2</td><td>C^2</td></tr>
<tr><td>1</td><td>1120000</td><td>3200</td><td>56000</td><td>4</td><td>490000</td><td>1600</td></tr>
<tr><td>2</td><td>6400000</td><td>16000</td><td>200000</td><td>6.25</td><td>1000000</td><td>6400</td></tr>
<tr><td>3</td><td>16940000</td><td>36300</td><td>462000</td><td>9</td><td>1960000</td><td>12100</td></tr>
</table>
Optimal parametric setting: $BC^2$3 $AC^2$3 ABC3 $A^2$3 $B^2$3 $C^2$3</td>
<td>Joint (TRS(WS)^2), ((TTA.(WS)^2), (TTA.TRS.WS), (TTA^2), (TRS^2), and (WS^2) yield (TRS(WS)^2)3 ((TTA.(WS)^2)3, (TTA.TRS.WS)3, (TTA^2)3, (TRS^2)3, (WS^2)3 to obtain 16940000 rpm(mm/min)^2, 36300 (mm/min)^2.^{0}), 462000 (rpm.mm/min.^{0}), 9(^{0}), 1960000 (rpm), 12100 (mm/min)^2.</td></tr>
<tr><td>Formula 79
<table>
<tr><td>Level</td><td>1/A</td><td>1/B</td><td>1/C</td></tr>
<tr><td>1</td><td>0.5</td><td>0.0014</td><td>0.025</td></tr>
<tr><td>2</td><td>0.4</td><td>0.001</td><td>0.0125</td></tr>
<tr><td>3</td><td>0.3333</td><td>0.0007</td><td>0.0091</td></tr>
</table>
Optimal parametric setting: 1/A1 1/B2 1/C1</td>
<td>Joint (1/TTA), (1/TRS) and (1/WS) yield (1/TTA)1, (1/TRS)2, (1/WS)1 to obtain 0.5 (1/^{0}), 0.001 (1/rpm), 0.025 (1/mm/min.).</td></tr>
<tr><td>Formula 80
<table>
<tr><td>Level</td><td>1/A</td><td>1/B</td><td>1/C</td><td>A</td><td>B</td><td>C</td></tr>
<tr><td>1</td><td>0.5</td><td>0.0014</td><td>0.025</td><td>2</td><td>700</td><td>40</td></tr>
<tr><td>2</td><td>0.4</td><td>0.001</td><td>0.0125</td><td>2.5</td><td>1000</td><td>80</td></tr>
<tr><td>3</td><td>0.3333</td><td>0.0007</td><td>0.0091</td><td>3</td><td>1400</td><td>110</td></tr>
</table>
Optimal parametric setting: 1/A3 1/B3 1/C3 A3 B3 C3</td>
<td>Joint (1/TTA), (1/TRS), (1/WS), (TTA), (TRS) and (WS) yield (1/TTA)3, (1/TRS)3, (1/WS)3, (TTA)3, (TRS)3, (WS)3 to obtain 0.333333 (1/^{0}), 0.000714 (1/rpm), 0.009091 (1/mm/min.), 3(^{0}), 1400 (rpm), 110 (mm/min).</td></tr>
<tr><td>Formula 81
<table>
<tr><td>Level</td><td>1/A</td><td>1/B</td><td>1/C</td><td>A^2</td><td>B^2</td><td>C^2</td></tr>
<tr><td>1</td><td>0.5</td><td>0.0014</td><td>0.025</td><td>4</td><td>490000</td><td>1600</td></tr>
<tr><td>2</td><td>0.4</td><td>0.001</td><td>0.0125</td><td>6.25</td><td>1000000</td><td>6400</td></tr>
<tr><td>3</td><td>0.3333</td><td>0.0007</td><td>0.0091</td><td>9</td><td>1960000</td><td>12100</td></tr>
</table>
Optimal parametric setting: 1/A3 1/B3 1/C3 $A^2$3 $B^2$3 $C^2$3</td>
<td>Joint (1/TTA), (1/TRS), (1/WS), (TTA^2), (TRS^2) and (WS^2) yield (1/TTA)3, (1/TRS)3, (1/WS)3, (TTA^2)3, (TRS^2)3, (WS^2)3 to obtain 0.333333 (1/^{0}), 0.000714 (1/rpm), 0.009091 (1/mm/min.), 3(^{0}), 1400 (rpm), 110 (mm/min), 9(^{0}), 1960000 (rpm), 12100 (mm/min)^2.</td></tr>
<tr><td>Formula 82
<table>
<tr><td>Level</td><td>BC/A</td><td>AC/B</td><td>AB/C</td></tr>
<tr><td>1</td><td>14000</td><td>0.1143</td><td>35</td></tr>
<tr><td>2</td><td>32000</td><td>0.2</td><td>31.25</td></tr>
<tr><td>3</td><td>51333.33</td><td>0.2357</td><td>38.1818</td></tr>
</table>
Optimal parametric setting: BC/A3 AC/B3 AB/C3</td>
<td>Joint (TRS.WS/TTA), ((TTA.WS)/TRS) and (TTA.TRS/WS) yield (TRS.WS/TTA)3, ((TTA.WS)/TRS)3, (TTA.TRS/WS)3 to obtain 51333.33 (rpm.mm/min. ^{0}), 0.235714 (^{0}.mm/min.rpm), 38.18182 (^{0}.rpm.min/mm).</td></tr>
<tr><td>Formula 83
<table>
<tr><td>Level</td><td>BC/A</td><td>AC/B</td><td>AB/C</td><td>A</td><td>B</td><td>C</td></tr>
<tr><td>1</td><td>14000</td><td>0.1143</td><td>35</td><td>2</td><td>700</td><td>40</td></tr>
<tr><td>2</td><td>32000</td><td>0.2</td><td>31.25</td><td>2.5</td><td>1000</td><td>80</td></tr>
<tr><td>3</td><td>51333.3333</td><td>0.2357</td><td>38.1818</td><td>3</td><td>1400</td><td>110</td></tr>
</table>
Optimal parametric setting: BC/A3 AC/B3 AB/C3 A3 B3 C3</td>
<td>Joint (TRS.WS/TTA), ((TTA.WS)/TRS) (TTA.TRS/WS), (TTA) (TRS) and (WS) yield (TRS.WS/TTA)3, ((TTA.WS)/TRS)3, (TTA.TRS/WS)3, (TTA)3, (TRS)3, (WS)3 to obtain 51333.33 (rpm.mm/min. ^{0}), 0.235714 (^{0}.mm/min.rpm), 38.18182 (^{0}.rpm.min/mm), 3(^{0}), 1400 (rpm), 110 (mm/min).</td></tr>
<tr><td>Formula 84
<table>
<tr><td>Level</td><td>BC/A</td><td>AC/B</td><td>AB/C</td><td>A^2</td><td>B^2</td><td>C^2</td></tr>
<tr><td>1</td><td>14000</td><td>0.1143</td><td>35</td><td>4</td><td>490000</td><td>1600</td></tr>
<tr><td>2</td><td>32000</td><td>0.2</td><td>31.25</td><td>6.25</td><td>1000000</td><td>6400</td></tr>
<tr><td>3</td><td>51333.3333</td><td>0.2357</td><td>38.1818</td><td>9</td><td>1960000</td><td>12100</td></tr>
</table>
Optimal parametric settings: BC/A3 AC/B1 AB/C1 $A^2$1 $B^2$3 $C^2$3; BC/A3 AC/B2 AB/C2 $A^2$2 $B^2$3 $C^2$3; BC/A3 AC/B3 AB/C3 $A^2$3 $B^2$3 $C^2$3</td>
<td>Joint (TRS.WS/TTA), (TTA.WS)/TRS) (TTA.TRS/WS), (TTA^2), (TRS^2) and (WS^2) yield (TRS.WS/TTA)3, ((TTA.WS)/TRS)1, (TTA.TRS/WS)1, (TTA^2)1, (TRS^2)3, (WS^2)3 to obtain 51333.33 (rpm.mm/min. ^{0}),</td></tr>
</table>

Formulation	Optimal parametric setting and interpretation
	0.114285714 (^{0}.mm/min.rpm), 35 (^{0}.rpm.min/mm), 4(^{0}), 1960000 (rpm), 12100 (mm/min)^2. Joint (TRS.WS/TTA), ((TTA.WS)/TRS) (TTA.TRS/WS), (TTA^2), (TRS^2) and (WS^2) yield (TRS.WS/TTA)3, ((TTA.WS)/TRS)2, (TTA.TRS/WS)2, (TTA^2)2, (TRS^2)3, (WS^2)3 to obtain 51333.33 (rpm.mm/min. ^{0}), 0.2 (^{0}.mm/min.rpm), 31.25 (^{0}.rpm.min/mm), 6.25 (^{0}), 1960000 (rpm), 12100 (mm/min)^2. Joint (TRS.WS/TTA), ((TTA.WS)/TRS) (TTA.TRS/WS), (TTA^2), (TRS^2) and (WS^2) yield (TRS.WS/TTA)3, ((TTA.WS)/TRS)3, (TTA.TRS/WS)3, (TTA^2)3, (TRS^2)3, (WS^2)3 to obtain 51333.33 (rpm.mm/min. ^{0}), 0.235714 (^{0}.mm/min.rpm), 38.18182 (^{0}.rpm.min/mm), 9(^{0}), 1960000 (rpm), 12100 (mm/min)^2.

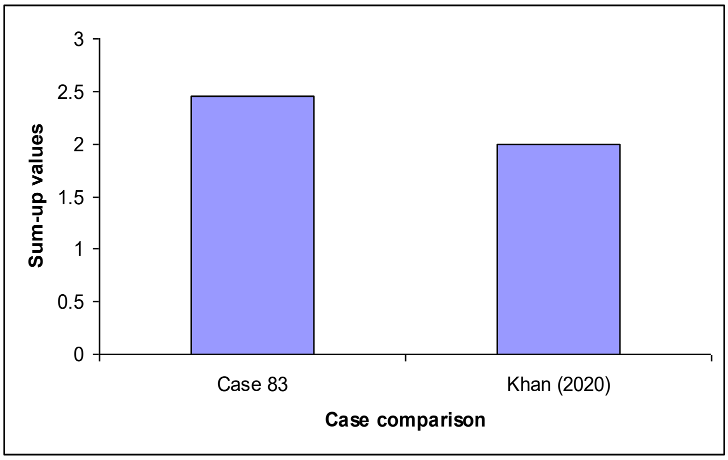

Figure 2. Comparison of the results of optimal the present study and Khan (2020)

It is important to compare the conventional results of optimal parametric setting and the results obtained in the present analysis using the 83 cases. This may be achieved by normalising the parameters using the initial format of the factor-level table and then considering the product of optimal parameters. Consider scenario 83, the factor-level table is adjusted to a normalized state where the aspect ratios are expressed between 0 and 1. In all the entries, the maximum and minimum normalized values are 0.60199 and 0.079602 when the linear normalisation method is used and the larger-the-better signal-to-noise criterion is used for all the aspect ratio parameters. Now, considering the first optimal parametric setting of BC/A$_3$ AC/B$_1$ AB/C$_1$ A$^2{}_1$ B$^2{}_3$ C$^2{}_3$, the values at each point are noted and added to the others and the overall value is noted. Here, the following are the yields of each of the optimal points: BC/A$_3$ = 0.527397, AC/B$_1$ = 0. 207818, AB/C$_1$= 0.335147, A$^2{}_1$= 0.207792, B$^2{}_3$ =0.568116 and C$^2{}_3$= 0.60199. These values sum up to 2.44826. However, to analyse the factor-level table data in Khan (2020), the normalised values are obtained to range from 0.173913 to 0.478261. Now, by considering the optimal points, the following are obtained: A$_3$ =0.4, B$_3$ = 0.451613 and C$_2$ = 0.347826. These values sum up to 1.99439. Figure 2 shows a comparison of the result of the conventional approach by Khan (2020) and the present study. From Figure 2, it is observed that scenario 83 provides a higher sum-up value of the optimal points of an optimal parametric setting, indicating a better method compared with the conventional method used in Khan (2020).

This study contributes to the existing literature on friction stir welding process parametric optimization by employing a novel approach of aspect ratios in the evaluation of the signal-to-noise ratios, which finally metamorphose into the response table that aids gives a head of how to interprete the optimal parametric setting, ranks and the delta values for the aspect ratios. Of particular significance is that a rigorous combination of aspect ratios expressed in powers of two is pursued. A complete divergence from the literature is pursued when only the aspect ratios are used for the determination of signal-to-noise ratios instead of a mixture of direct parameters and aspect ratio parameters. On the other side, the study aligns with the established practices in the Taguchi methodical optimization determination by following the route of first obtaining a combination of factors and levels, establishing their orthogonal matrices, calculating their signal-to-noise ratios, finding their averages and then stating the optimal parametric settings, ranks and delta values. Thus, the findings add to the body of knowledge on the Taguchi method by arguing that aspect ratios have impacts on the results of optimization. Consequently, by reflecting on the current state of knowledge on the friction stir welding literature, the paper strengthens the contribution made by parameters towards establishing optimal thresholds of the friction stir welding process. The work also offers valuable understanding for process engineers and general managers of engineering work centres that develop and implement welding policies toward the deployment of a successful value-adding welding practice in the work centre.

5. CONCLUSIONS

This study examines how the optimal thresholds of parameters critical to friction stir welding, in the welding of the AA606Z-T6 alloy, can be determined through a modified Taguchi method. The modification to the Taguchi method was achieved by paying attention to the parameters. In a divergent view from the literature, aspect ratios were used to replace the traditional parameters that have been used in most studies on the Taguchi method to date. The uniqueness of the approach is that the work considers the most possible formulation where direct parameters are considered with aspect ratios. In a formulation of 83 different computational platforms for the Taguchi scheme, an overwhelming majority consists of aspect ratios.

Interestingly, at a variable from the traditional layout of the factor-level table where each parameter occupies a column before deciding on the suitable orthogonal matrix for evaluation, products of parameters that may replace a single parameter-squares of a parameter were also substituted for a parameter. The intention was to test the widest possible extent to which the parameters could be combined to develop an orthogonal matrix. Based on the analysis and the findings presented in this study, the following conclusions could be drawn from the present study:

1. By representing a parameter along the column of a factor-level framework, the simplest framework to determine the orthogonal array may be established. However, the performance of the Taguchi method regarding the optimal parametric settings, ranks and delta values is not as comprehensive as when aspect ratios are considered to replace individual parameters.
2. The aspect ratios as parameters are easy to use and give more assurance of a reliable measure of optimization than direct parameters.

There are opportunities for future studies when the present study is extended in many circumstances. The use of aspect ratios may be made when the Taguchi method is merged with other methods such as the regression method and exponential smoothing for casting methods. Furthermore, we have restricted the present study to low powers of two. However, higher powers of three or more could be tested and comparisons made with the results obtained in the present study. Also, multiple parameters may be multiplied as a parameter where each column has the highest number of multiplications of parameters (i.e. ABCD, which is 4). The next column may be reduced by parameters, i.e. ABC, which is 3). Then others could be ABC and the last one, A. This pattern is unprecedented in the aspect ratio research and could be tested. Notwithstanding, it may involve rigorous computations that could be challenging to make without computational facilities. Future studies could also pursue sensitivity tests of the aspect ratio parameters.

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